RootLocus.exe

User's Guide

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TRANSLATION: YOU ASSUME ALL LIABILITY WHEN YOU USE ROOTLOCUS.EXE.

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FEATURES

- RootLocus makes computing, plotting, and evaluating a root locus easy for both the practicing engineer and the engineering student
- RootLocus computes and plots a root locus using a linear system's open-loop transfer function that you provide
- RootLocus enables you to step through the plotted locus for each computed gain value and automatically identify the points on the locus associated with each gain. The gain and complex coordinates of the computed poles of the identified locus points are listed simultaneously.
- RootLocus plots the step response for the selected gain and calculated pole positions
- RootLocus plots the frequency response for the selected gain and calculated pole positions
- RootLocus automatically calculates the damping factor under certain conditions as you step through the plotted locus for each computed gain value
- RootLocus plots the Bode amplitude and phase responses for the open-loop pole positions
- RootLocus.exe is a free-standing executable file. It needs no installation. Simply copy it onto a Windows system and run it!

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SYSTEM REQUIREMENTS

- RootLocus is for use on Windows systems running Windows 10 or 11
- The computer monitor and video driver must be able to accommodate the large RootLocus form which has dimensions of 887 pixels wide by 647 pixels high

SUGGESTED REQUIREMENTS

- You should have a basic knowledge of complex numbers and the complex plane
- You should have a basic knowledge of Laplace transforms and transfer functions
- You should have a basic knowledge of linear control systems
- Although a brief explanation of the root locus method is provided, both here and in the Help menu, it is helpful—but not necessary—for you to have had an introduction to the root locus method

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PROVIDE FEEDBACK TO THE AUTHOR

To provide feedback to the author, send e-mail to

rootlocus@bobpenoyer.com

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INSTALLATION

File ROOTLOCUS.EXE needs no installation. Simply copy the file to a Windows system. Then double-click on the file name or icon to run the program.

RootLocus is for use on Microsoft Windows® systems running Windows 10 or 11. RootLocus might function correctly on earlier versions of 32-bit Windows but this has not been verified.

THE ROOT LOCUS METHOD IN BRIEF

The root locus method is a clear, simple, graphical technique for evaluating a control loop's stability and behavior. The root locus method predicts a control system's closed-loop behavior and stability based upon the system's open-loop transfer function and loop gain.

The basic concept of the root locus is that the poles of a linear closed-loop control system move as the loop gain changes. The paths of these poles are plotted on the complex plane and form a locus of points, the root locus. If the root locus never crosses into the right-half plane (i.e., to the right of the imaginary, or jω, axis) the loop is stable. If any poles cross into the right-half plane, the loop can be unstable.

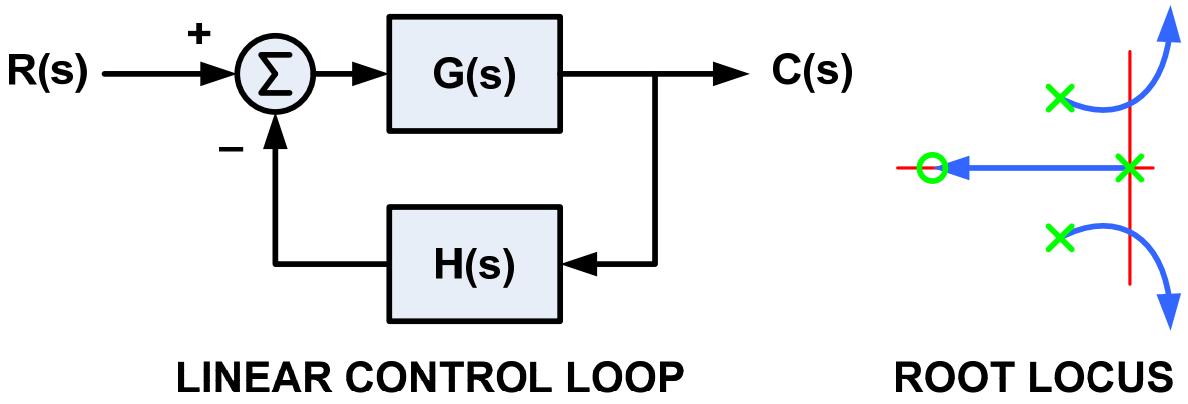

Figure 1. A typical linear control system and an example of a root locus

In addition to stability, the root locus graphically demonstrates a control loop's damping factor, rate of settling, damped natural frequency (frequency of ringing,) and undamped natural frequency.

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The root locus method for the analysis of linear closed-loop systems was developed by W.R. Evans and first published by him in 1948. Though there are many excellent references for the root locus method, one presentation by Evans, himself, is "Control-system Dynamics," McGraw-Hill Book Company, Inc., New York, 1954.

BETTER THAN BODE PLOTS

The Bode plot method analyzes a linear control system for a single value of gain. A control loop's gain margin and phase margin reveal if the loop is stable *for that gain only!* Nevertheless, RootLocus can calculate and plot Bode plots.

THE POWER OF THE ROOT LOCUS METHOD

The root locus method, on the other hand, readily permits a system's characteristics to be analyzed for ALL values of gain. So it's easy to tell if there is any possibility that the loop can become unstable. The RootLocus program makes computing, plotting, and analyzing the root locus very easy.

A control system might oscillate or be unstable for some discrete value(s) of gain or some range(s) of gain. The root locus method permits easy identification of these possibilities. This is the power of the root locus method.

THE LINEAR CLOSED-LOOP SYSTEM

The *closed-loop transfer function* of the system of Figure 1 is

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} \qquad \text{Eq. 1}$$

The *characteristic equation* of the system is

$$1 + G(s)H(s) = 0 \qquad \text{Eq. 2}$$

Note that if the negative sign in Figure 1 is changed to a positive sign (i.e., positive feedback,) the *plus* symbol in each of Eq. 1 and Eq. 2 is changed to a *minus*.

More generally, a pure gain term is presumed to exist somewhere in the closed-loop path of Figure 1, either in the forward path, cascaded with G(s), or in the feedback path, cascaded with H(s). Figure 2 shows the added gain term in two different places in the loop. Either way, the closed-loop transfer function becomes

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$$\frac{C(s)}{R(s)} = \frac{K\,G(s)}{1 + K\,G(s)H(s)} \quad \text{or} \quad \frac{C(s)}{R(s)} = \frac{G(s)}{1 + K\,G(s)H(s)} \qquad \text{Eq. 3}$$

So, the characteristic equation becomes

$$1 + K\,G(s)H(s) = 0 \qquad \text{Eq. 4}$$

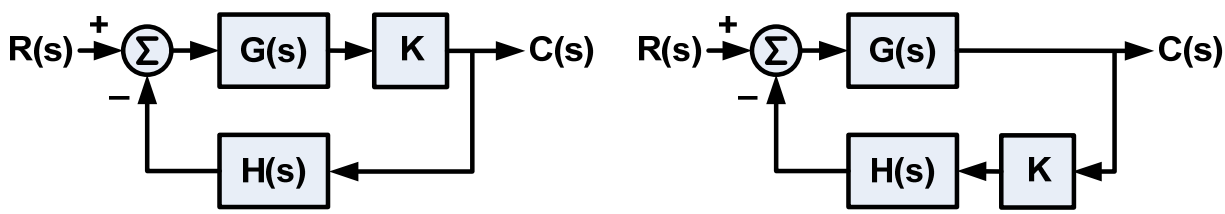

Figure 2. The control loop is generalized by inserting a pure gain term anywhere in the loop

Variable K in Eq. 4 may be positive or negative. A positive K implies negative feedback while a negative K implies positive feedback. *K has magnitude and sign only and contributes nothing to the overall phase of the system aside from its sign.* Only G(s) and/or H(s) contribute phase shifts to the system.

Temperature, aging, component value differences from system to system, and other conditions can cause K's magnitude to vary. K may vary simply because a gain control setting is changed.

It should be apparent from Eq. 4 that the roots of the characteristic equation change as K is allowed to change.

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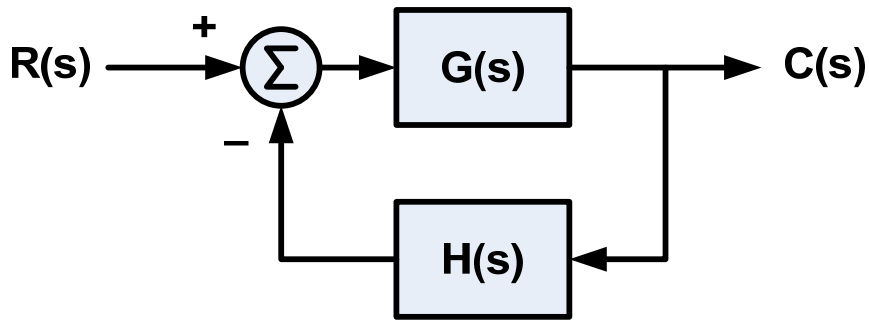

Figure 3. A closed-loop linear control system

The system in Figure 3 has the following characteristics.

Open-loop transfer function: $\boxed{G(s)H(s)}$ Eq. 5

The open-loop transfer function is the effect of G and H on the input signal as the signal passes through the loop. The open-loop transfer function is the basis for the root locus method.

Closed-loop transfer function: $\boxed{\dfrac{C(s)}{R(s)} = \dfrac{G(s)}{1+G(s)H(s)}}$ Eq. 6

Not surprisingly, the closed-loop transfer function represents the behavior of the closed loop. It is this behavior that is evaluated by the root locus method by using the open-loop transfer function.

General closed-loop transfer function:

$$\boxed{\frac{C(s)}{R(s)} = \frac{K\,G(s)}{1+K\,G(s)H(s)}} \quad \text{or} \quad \boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{1+K\,G(s)H(s)}} \qquad \text{Eq. 7}$$

The general closed-loop transfer function of Eq. 7 is the function that would be obtained if a gain of K is cascaded with G or H in Figure 3. This was illustrated in Figure 2. The use of an arbitrary gain facilitates the development of the root locus method.

ZEROS

The zeros of the linear system are the complex roots of the *numerator* of Eq. 5.

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Eq. 5 is the open-loop transfer function. That is, it represents the characteristics of the "loop" when the loop is opened.

The positions of the zeros on the complex plane never change.

POLES

The poles of the linear system are the complex roots of the general characteristic equation,

$$1 + K\,G(s)H(s) = 0 \qquad \text{Eq. 8}$$

K, a pure gain coefficient, is assumed to be cascaded with G(s) or H(s), or distributed between them, to account for any gain variations in G and/or H. *As K varies, the positions of the poles on the complex plane vary.*

As the positions of the poles vary, i.e., as the poles migrate on the complex plane, they describe a locus—the *root locus*.

MIGRATING POLES FORM THE ROOT LOCUS

The poles always start at their open-loop locations on the complex plane.

The open-loop poles are given by the roots of the *denominator* of the open-loop function

$$G(s)H(s) \qquad \text{Eq. 9}$$

The poles of

$$1 + K\,G(s)H(s) = 0 \qquad \text{Eq. 10}$$

always migrate on the complex plane from their starting points when K = 0 (i.e., from the open-loop poles) toward the zeros as K approaches infinity.

When an insufficient number of zeros exist as destinations for the poles (on a one-for-one basis,) the remaining poles migrate toward infinity. *The paths the loci follow depend upon the nature of the system and whether K is positive or negative.*

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LOCATING THE ROOT LOCUS ON THE COMPLEX PLANE

The RootLocus program computes and plots the locus for you, so the description that follows is merely an explanation to provide a clearer understanding of the details of a root locus.

The root locus exists anywhere on the complex plane (i.e., at any complex value of s) where the characteristic equation is true. If Eq. 10, the general characteristic equation, is rewritten as

$$G(s)H(s) = -\frac{1}{K} \qquad \text{Eq. 11}$$

then it becomes more clear that when the MAGNITUDE CONDITION,

$$|G(s)H(s)| = \left|\frac{1}{K}\right| \qquad \text{Eq. 12}$$

and the PHASE CONDITION (in degrees),

$\angle[G(s)H(s)] = \pm 180°(2n+1)$ for positive K, or

$\angle[G(s)H(s)] = \pm 360° n$ for negative K

(n = any integer) are met simultaneously, the locus exists there.

Clearly, as the value of K varies, the magnitude condition also varies. The poles migrate along the locus as K varies.

The need to use the magnitude and phase conditions to locate the locus on the complex plane is eliminated by using high-speed computers to simply solve for the roots of the characteristic equation for any given value of K and over a range of Ks. This is what the RootLocus program does.

THE LEFT-HALF AND RIGHT-HALF PLANES

The left-half plane is everything to the left of the imaginary, or jω, axis. So, of course, the right-half plane is everything to the right of the jω axis. One of the most important aspects of the root locus method is the distinction between the left- and right-half planes. Why?

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Take a second-order system for example. A second-order transfer function might have the form given in Eq. 13.

$$\frac{C(s)}{R(s)} = \frac{\omega}{(s+\sigma)^2 + \omega^2} \qquad \text{Eq. 13}$$

The roots of the denominator of Eq. 13 are

$$s = -\sigma \pm j\omega \qquad \text{Eq. 14}$$

The inverse Laplace transform of the right side of Eq. 13 is

$$e^{-\sigma t} \sin \omega t \qquad \text{Eq. 15}$$

Eq. 15 represents a function that rings at a frequency of ω = 2πf and decays (i.e., reduces its amplitude) as a function of $e^{-\sigma t}$. You should understand that the -σt term causes the value of $e^{-\sigma t}$ to become smaller as time, t, increases.

Look at Eq. 14. It represents a pair of poles on the complex plane in the left-half plane as shown in Figure 4.

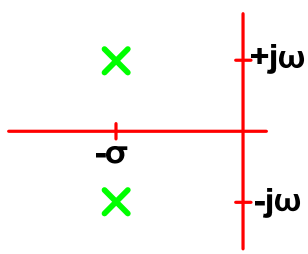

Figure 4. Complex poles in the left-half plane

Eq. 14 and Eq. 15 represent the same thing: a function that rings but decays with increasing time. *Complex poles in the left-half plane ring and decay. This is a stable system.*

Consider what happens when the sign in front of σ in Eq. 13 is negative. In that case, Eq. 14 becomes

$$s = \sigma \pm j\omega \qquad \text{Eq. 16}$$

And Eq. 15 becomes $$e^{\sigma t} \sin \omega t \qquad \text{Eq. 17}$$

Eq. 16 represents a pair of poles in the right-half plane as shown in Figure 5.

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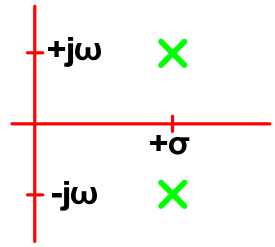

Figure 5. Complex poles in the right-half plane

Eq. 17 represents a function that rings at a frequency of ω = 2πf with an amplitude that *increases exponentially without bound*. You should understand that the *+σt* term causes the value of $e^{\sigma t}$ to become larger as time, t, increases. *Complex poles in the right-half plane ring and increase without bound. This is an unstable system!*

Finally, when σ in all of the previous equations goes to zero, we have

$$s = \pm j\omega \qquad \text{Eq. 18}$$

and

$$\sin \omega t \qquad \text{Eq. 19}$$

Since σ = 0, the poles are not in the left- or right-half planes. Instead, they are on the imaginary, or jω, axis as shown in Figure 6.

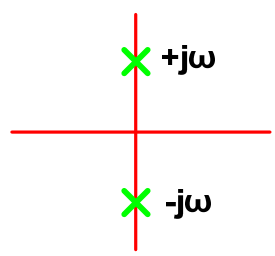

Figure 6. Complex poles on the jω axis

This is pure oscillation with no decay or increase of amplitude. Complex poles on the jω axis represent an oscillating system. *Complex poles on the jω axis oscillate forever. This is an unstable system!*

These results provide the most fundamental root locus rule:

> ***The value and sign of the real part of the poles determine a linear control system's stability.***

Variable s is complex with the form s = σ + jω.

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- **When σ is negative, the system is stable** since the amplitude of any ringing will decay to zero.

- **When σ is positive, the system is unstable** since the amplitude of any ringing will increase without bound.

- **When σ = 0, the system is unstable** since the ringing will continue at a frequency of $\omega = 2\pi f$ forever with no change of amplitude.

You might ask, What happens when $\omega = 0$, i.e., when you simply have $s = \sigma + j0$? This equation represents a condition with a pole on the real axis. When σ is negative, any DC offset will settle toward 0, just as complex poles in the left-half plane ring but settle to an amplitude of 0. When σ is positive, any DC offset will ramp up exponentially to infinity, just as complex poles in the right-half plane ring and increase without bound. When $\sigma = 0$, any DC offset will remain unchanged.

BASIC ROOT LOCUS RULES

The following are some of the most basic and useful root locus rules. While additional rules can be found in references on control systems, this list gives you "the most bang for the buck."

1. When, for a given value of K, *all* of the poles reside to the left of the imaginary ($j\omega$) axis, the system is stable.

2. When, for a given value of K, *any* poles reside to the right of the imaginary ($j\omega$) axis, the system is unstable.

3. When the entire locus resides to the left of the imaginary axis, the system is *unconditionally stable*.

4. If any part of the locus exists on the right-half plane for some value(s) or range(s) of K, but all of the migrating poles exist in the left-half plan for other values of K, the system is *conditionally stable*.

5. When, for a given value of K, a complex conjugate pair of poles resides on the imaginary ($j\omega$) axis at $\pm j\omega$, the system will oscillate at a frequency of $\omega = 2\pi f$, where $f = \omega / 2\pi$ is frequency in hertz. Stated another way, the system will oscillate for the gain and at the frequency where the locus crosses the imaginary axis; i.e., if the locus touches or crosses the imaginary axis at any point, the system can oscillate there.

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6. In a second-order system, or a system with dominant poles (i.e., poles that are significantly closer to the imaginary axis than any other poles) that behave like a second-order system, *the cosine of the angle from the negative real axis to the line extending from the origin (0 + j0) to one dominant complex pole is the damping factor for the system.*

CHARACTERISTICS OF A TYPICAL ROOT LOCUS PLOT

1. *A circle, or "O", identifies the location of each zero.* When multiple zeros reside at the same location, the point is identified by a single "O".

2. *A cross, or "X", identifies the starting point of each pole*, i.e., the location of each pole for K = 0. *K = 0 corresponds to the open-loop condition.* When multiple poles originate from the same location, that location is identified by a single "X".

3. Some other symbol, or a smooth curve, is used to mark the path of the migrating poles as K varies from 0 to infinity.

MIGRATING POLES IN THE RootLocus PROGRAM

The RootLocus program uses a dot to represent the position of each of the computed poles when K ≠ 0. The arrangement of the dots tends to suggest the curve, or locus, of the root locus. The dots often run together to form a continuous curve.

The RootLocus program uses squares that are larger and a different color than the locus dots to indicate the pole positions for any selected gain. When the gain is varied in the RootLocus program, the squares move along the set of dots, or the locus, to show where the poles exist for each gain.

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A ROOT LOCUS EXAMPLE

Suppose a system has an open-loop transfer function of

$$\boxed{G(s)H(s) = \frac{(s+20)^2}{s\left(s^2+0.1s+50\right)} = \frac{s^2+40s+400}{s^3+0.1s^2+50s}}$$

Then the open-loop zeros are the roots of

$$(s+20)^2 = 0$$

Clearly, there are two zeros, both located at s = -20. Or, when s is treated as a general complex number, both zeros are located at s = -20 + j0.

The open-loop poles are the roots of

$$s\left(s^2+0.1s+50\right) = 0$$

It should be clear from the above equation that a pole exists at s = 0. Or, as a complex number, s = 0 + j0.

There are two more poles due to $s^2+0.1s+50=0$

The roots of this function are complex. In this case, s = -0.05 ± j7.0709.

The poles that have just been found are the ***open-loop poles****.* Careful examination of Eq. 10 reveals that it can be written in terms of the numerator and denominator as

$$1+\frac{K \times num[G(s)H(s)]}{den[G(s)H(s)]} = 0$$

or,

$$den[G(s)H(s)] + K \times num[G(s)H(s)] = 0$$

So when K = 0, you are left with $den[G(s)H(s)] = 0$

But the roots of the last equation are simply the poles of the open-loop equation! That is, *the poles of the open-loop equation represent the starting points of the poles, i.e., where K = 0.*

From Eq. 11,

$$G(s)H(s) = -\frac{1}{K}$$

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we can see that $$\lim_{K\to\infty} G(s)H(s) = 0$$

But the last equation goes to 0 when its numerator goes to 0—and the roots of the numerator are the zeros! Hence, *the poles (i.e., the roots of the characteristic equation) approach the zeros as K goes to infinity.*

If there aren't enough zeros for each of the poles to migrate toward, on a one-for-one basis, the remaining poles simply go to infinity.

Figure 7 was plotted by the RootLocus program. It shows the green zeros and poles of the example open-loop transfer function. It also shows the blue loci dots of the closed-loop transfer function. The individual dots mark the points where individual closed-loop poles were calculated as K was varied over a range of K = 0 to K = 150.

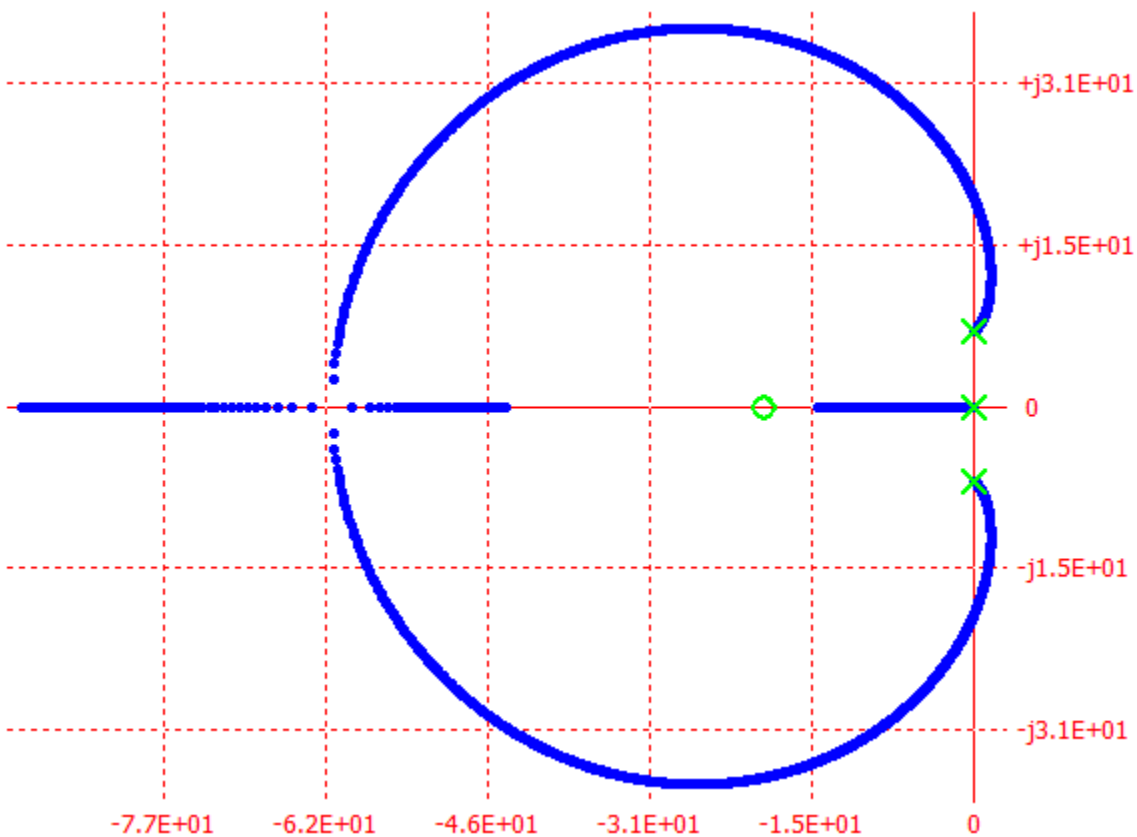

Figure 7. An example of a root locus

Upon completing the calculations and plotting the locus of Figure 7, RootLocus also lists these results:

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```
ZEROS:

   REAL         IMAG
-2.0000E+01 +0.0000E+00
-2.0000E+01 +0.0000E+00

OPEN-LOOP POLES:

   REAL         IMAG
+0.0000E+00 +0.0000E+00
-5.0000E-02 +7.0709E+00
-5.0000E-02 -7.0709E+00
```

Figure 8 highlights some of the details of the root locus in Figure 7.

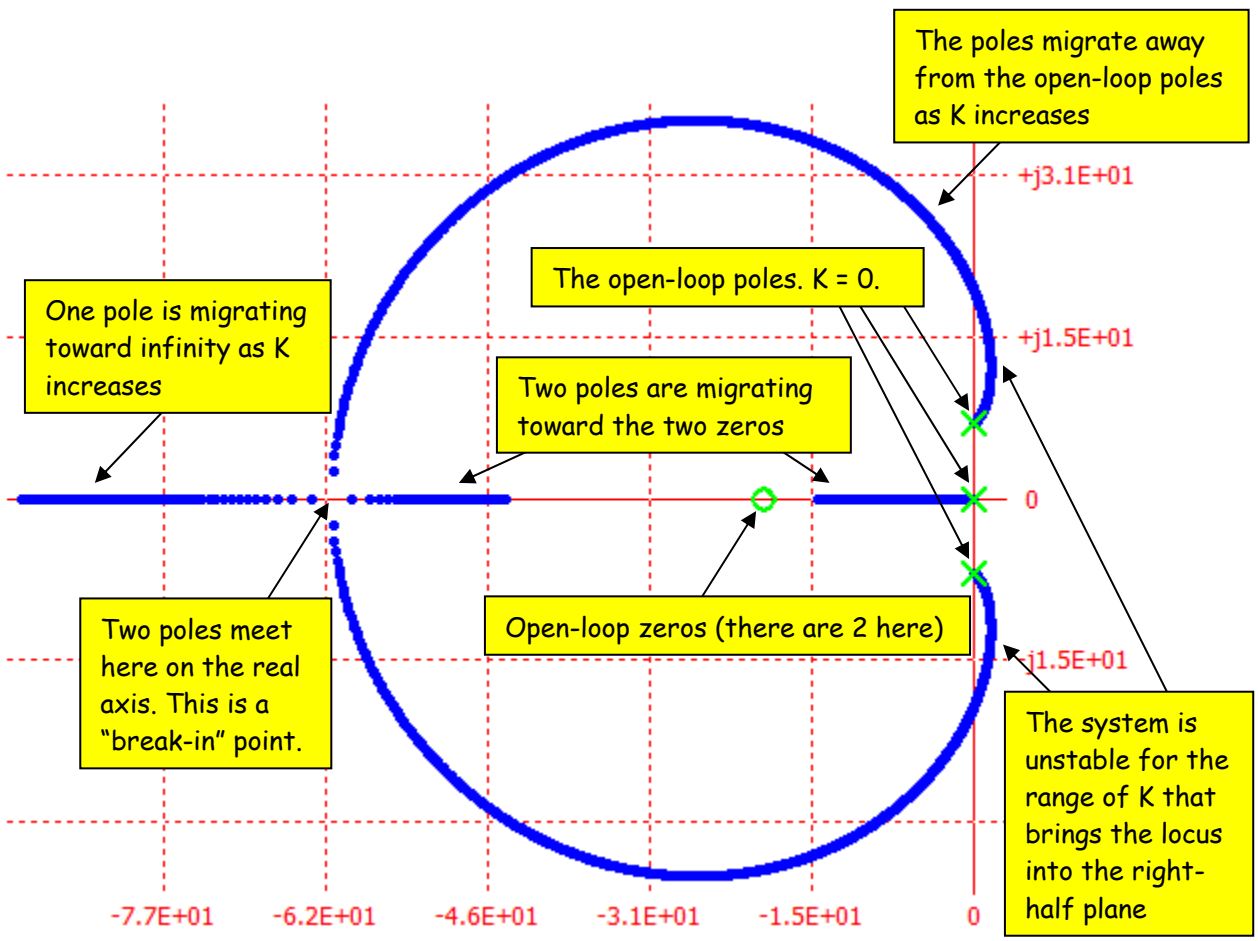

Figure 8. Some of the parts of a root locus

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It's easy to see from only a brief look at Figure 8 how the example system behaves for *all* values of K. Whereas a Bode plot shows how a system behaves for only a single value of K, a root locus, like the one in Figure 8, quickly shows that the example system is unstable for some range of K and stable elsewhere. That is, the example system is *conditionally stable*.

Question: ***For what values of K is the example system stable or unstable?***

RootLocus answers the question easily. Once RootLocus calculates and plots the root locus, it permits the user to vary K easily over its entire range by moving the slider on a scrollbar. RootLocus highlights the poles for each chosen value of K, making it easy to identify their locations. Simply sliding the scrollbar that is provided for this purpose allows K to vary over the entire range that was chosen for the calculation.

Sliding the scrollbar reveals the results in Figure 9. The figure shows the poles that RootLocus has highlighted as the locus begins to move into the right-half plane. Notice that the real parts of the complex pole pair have taken on a positive sign. So beginning at about K = 0.0152, the system becomes unstable.

The exact value of K generally cannot be found for the point where the locus crosses the imaginary axis. This is due to the fact that RootLocus calculates the locus by using arbitrary, discrete increments of gain. However, by choosing the plotting density finely enough, a good estimate of the beginning of the unstable range of K can be determined.

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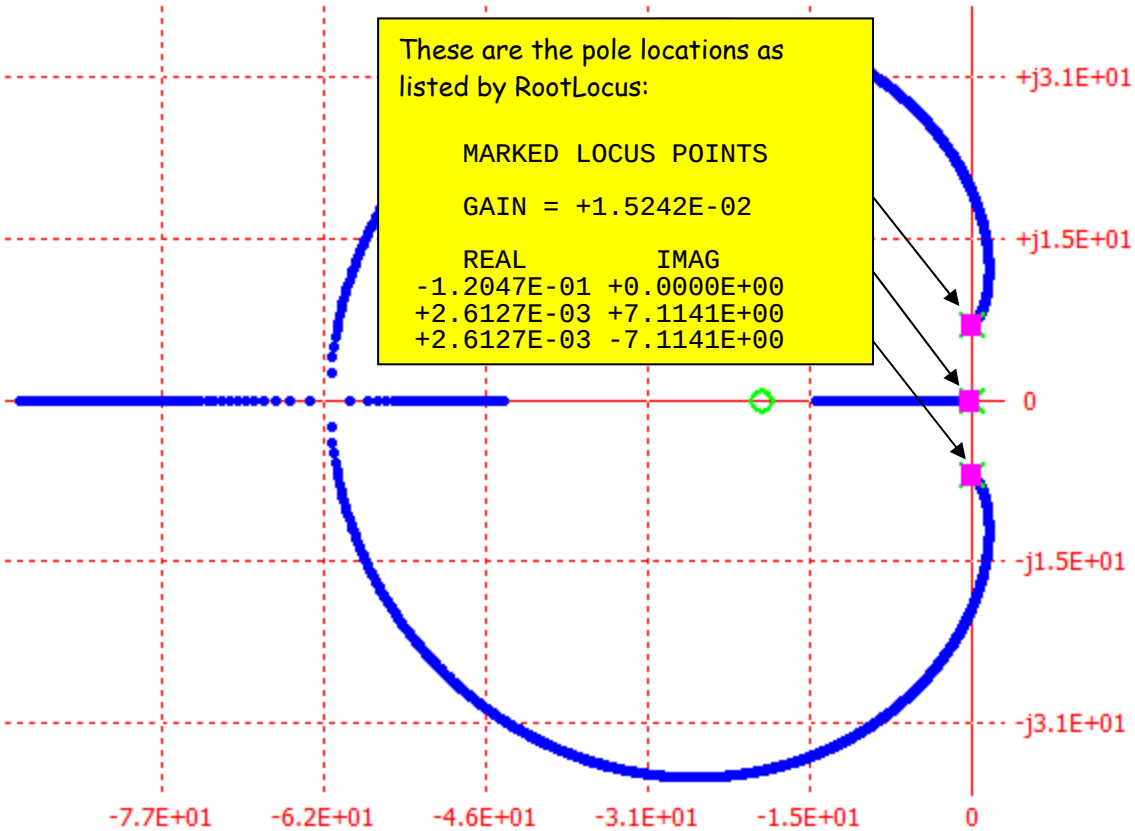

Figure 9. The computed poles begin moving into the right-half plane when K = 0.0152

By sliding the scrollbar to increase gain (K) further, the point can be found where the locus again returns to the left-half plane. *When all of the poles reside in the left-half plane, the system is stable.*

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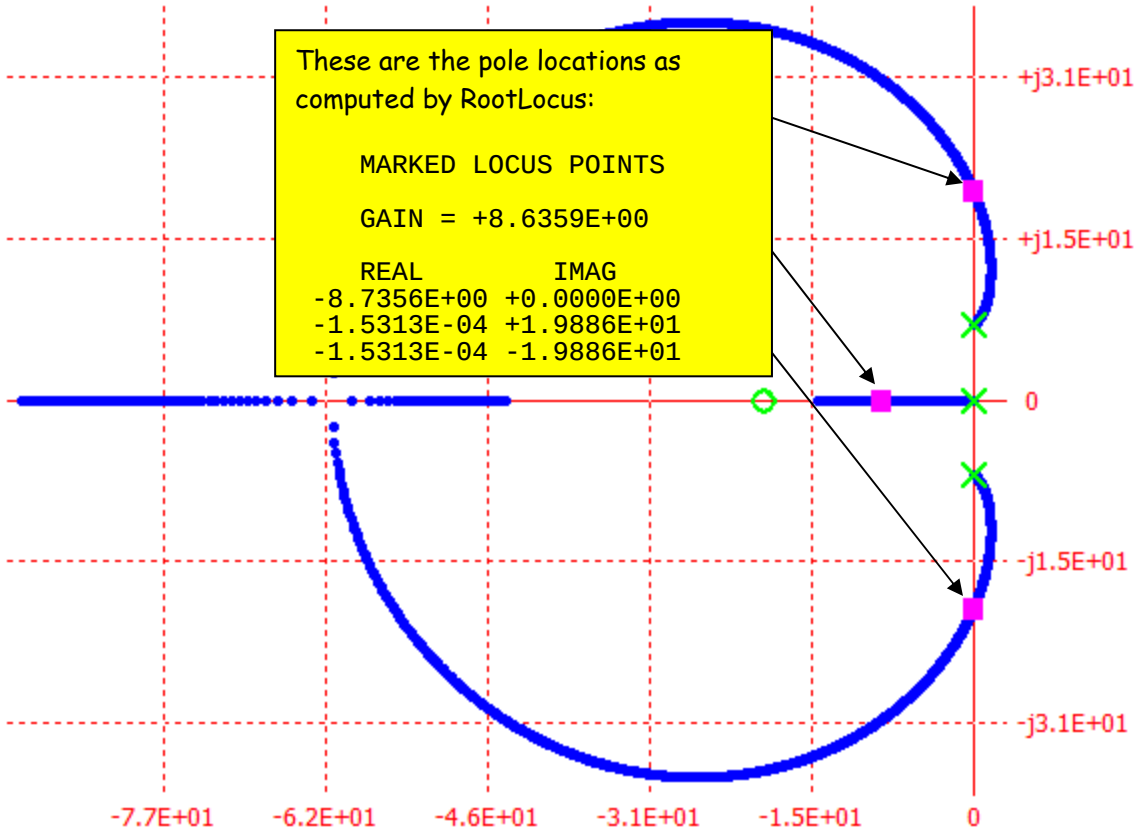

Figure 10. The computed poles begin moving into the left-half plane when K = 8.64

Figure 10 shows that with a gain of about K = 8.64, the poles have returned to the left-half plane (i.e., the real parts the poles are now all negative) and the loop is stable here.

The question has now been answered for the example system: *When K is maintained below about 0.0152 or above about 8.64 the loop is stable.* These are the conditions for stability.

The root locus method and the computational facilities provided by RootLocus make questions about the stability of linear control systems easy to resolve.

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USING RootLocus

WHEN RootLocus OPENS

Numerator Coefficients

When RootLocus opens, it contains an example open-loop transfer function. These numerator coefficients are entered:

1 20

The coefficients are ordered in highest-to-lowest order so that these numerator coefficients represent

$$s + 20$$

Coefficients must be entered as numbers separated by at least one space. Legal characters include the digits 0-9, "+" or "-" signs, and either "e" or "E". These are examples of legal entries:

1 200 3000 -5

1 +2e2 3E3 -5E0

Coefficients must be entered as the coefficients of expanded polynomials, not as factored polynomials. So expand terms such as

$$(s+6)(s-2)^3$$

into $$s^4 - 24s^2 + 64s - 48$$

then enter the coefficients as 1 -24 64 -48

The numerator can have a minimum order of 0 or a maximum order of 20. RootLocus parses the coefficients to create the correct polynomial. However, *the parsing algorithm is not robust, so care is needed to make any entry conform to the formats described here.*

Denominator Coefficients

When RootLocus opens, it contains an example open-loop transfer function. These denominator coefficients are entered:

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1 0.1 5

The coefficients are ordered in highest-to-lowest order so that these denominator coefficients represent

$$s^2 + 0.1s + 5$$

The denominator can have a minimum order of 1 or a maximum order of 20. RootLocus parses the coefficients to create the correct polynomial. However, *the parsing algorithm is not robust, so care is needed to make any entry conform to the format described here.*

The denominator order cannot be less than the numerator order. RootLocus will flag an error if the denominator order is too small.

<u>Gain Limits</u>

When RootLocus opens, it provides default Maximum Gain = 100 and Minimum Gain = 0.

Any range of gain that is selected can include 0 but cannot pass through 0. Also, any gain range must be all non-negative (i.e., any range from 0 through some positive limit) or all non-positive (i.e., any range from 0 through some negative limit.)

It is not possible, for example, to declare a Minimum Gain of -100 and a Maximum Gain of +100 since the range passes through 0.

However, it is possible to declare a Minimum Gain of -100 and a Maximum Gain of 0, or -1, or -10, or any non-positive number—as long as the Maximum Gain is more positive than the Minimum Gain. For example,

Legal gain limits:	Maximum Gain = 10000	Positive Gain Limits
	Minimum Gain = 10	
Legal gain limits:	Maximum Gain = 1000	Positive Gain Limits
	Minimum Gain = 0	

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Legal gain limits:	Maximum Gain = 0	Negative Gain Limits
	Minimum Gain = -100	
Legal gain limits:	Maximum Gain = -25	Negative Gain Limits
	Minimum Gain = -420	
Illegal gain limits:	Maximum Gain = 100	Illegal Limits
	Minimum Gain = 200	
Illegal gain limits:	Maximum Gain = 100	Illegal Limits
	Minimum Gain = -100	

Plotting Density

When RootLocus opens, it provides a default plotting density of *Normal*. Seven choices are possible. The choice you use depends upon the general appearance of the computed locus and the limits of the capabilities of RootLocus.

RootLocus calculates a maximum of 50,000 roots, or pole values. If RootLocus determines that the computation will require more than this number, it prompts you to select a coarser plotting density or reduce the gain range. If the coarsest density had been chosen, you will be prompted to decrease the gain range.

Plotting Color

When RootLocus opens, it provides a default plotting color of Normal Colors. You are also provided with a choice of Black.

Black might be the better choice for plotting if you intend to copy the root locus into a document that will be printed in black and white.

Black might be a better choice for any user who is color blind and for whom the normal plotting colors are difficult to see.

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CALCULATING THE ROOT LOCUS

Calculate

If RootLocus is already open, close it. Now open RootLocus to recover the default starting parameter values. Using the default conditions, click on the Calculate button. The root locus for the default coefficients appears in the plotting area. It should look like Figure 11.

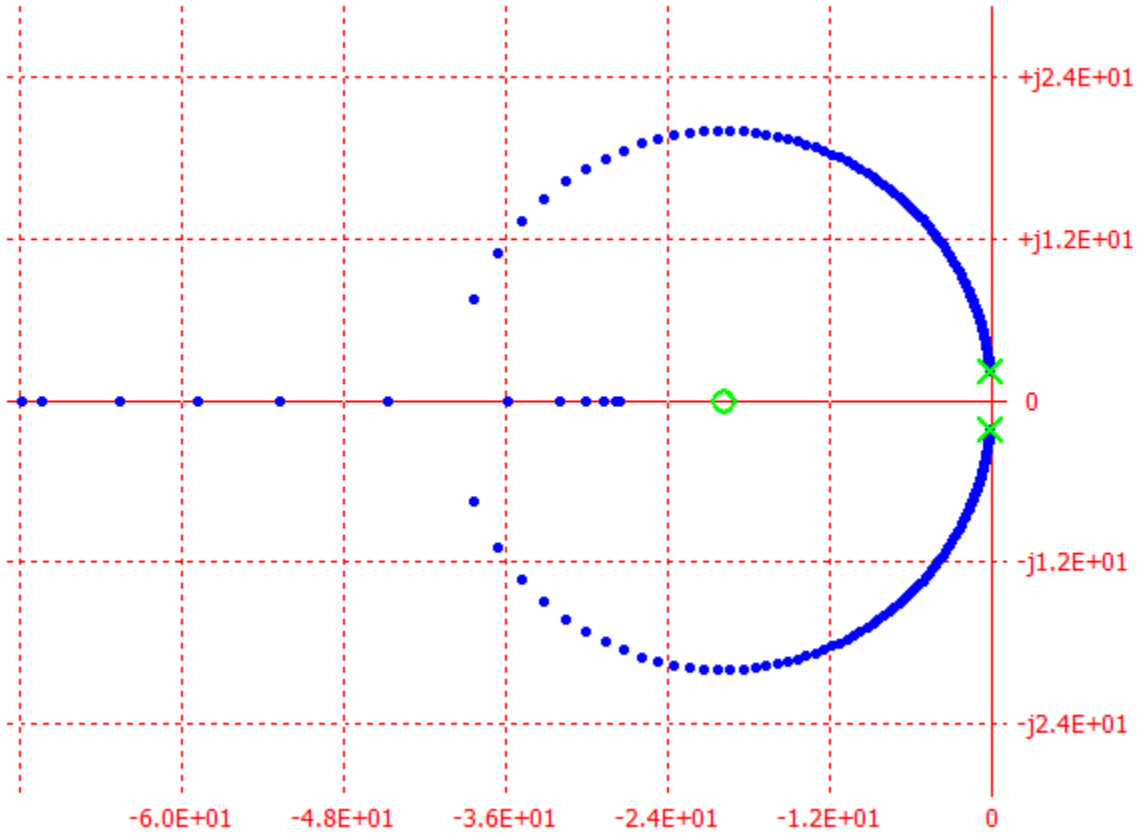

Figure 11. Root locus for the default coefficients, gain range, and plotting density

The calculated zeros and open-loop poles are listed in the panel at the right. They are:

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```
ZEROS:

   REAL        IMAG
-2.0000E+01 +0.0000E+00

OPEN-LOOP POLES:

   REAL        IMAG
-5.0000E-02 +2.2355E+00
-5.0000E-02 -2.2355E+00


<--- Use the scrollbar
     to scan through all
     gains
```

Notice the information at the bottom of the list. It tells you that you can now use the scrollbar to view the poles at each calculated gain (K) from the default Minimum Gain of 0 to the default Maximum Gain of 100.

Slide the scrollbar until you reach a gain of GAIN = +6.9957E+01 as listed in the right panel. The plot should now look like Figure 12.

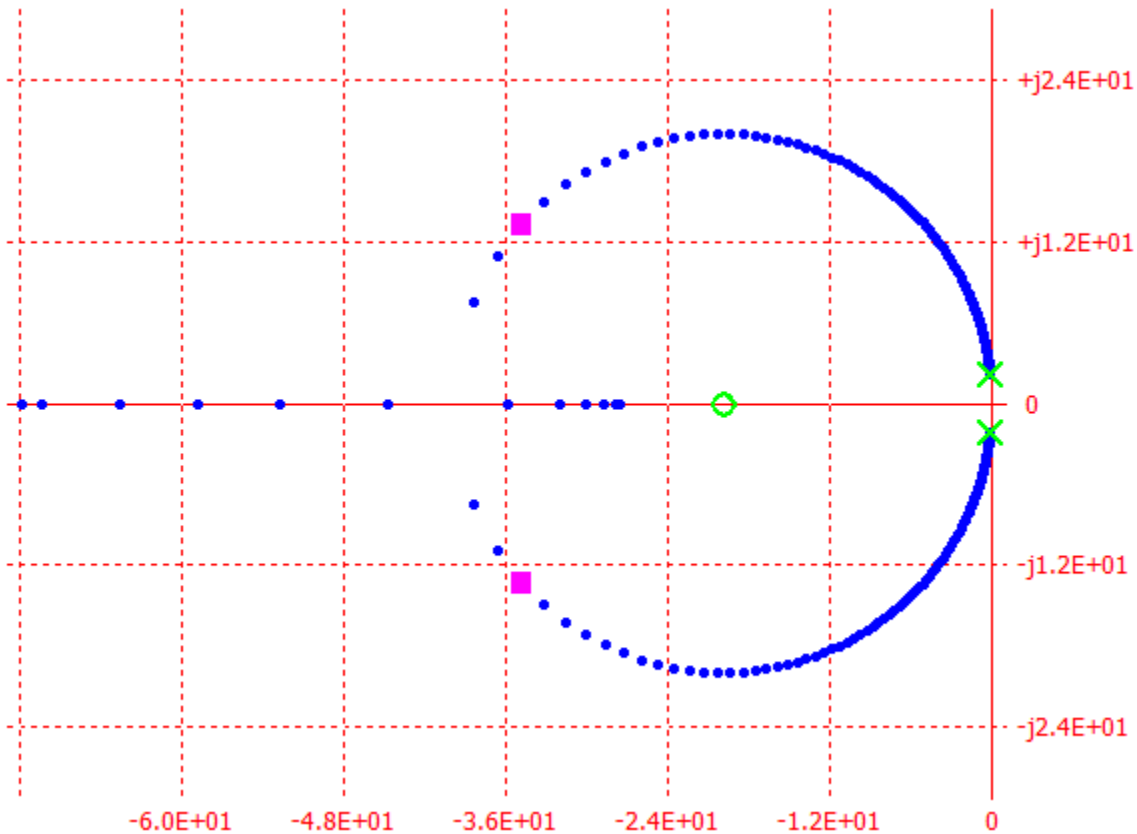

Figure 12. Highlighted poles for GAIN = +7.0070E+01

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The listed results for Figure 12 are

```
MARKED LOCUS POINTS

GAIN = +6.9957E+01

REAL        IMAG
-3.5029E+01 +1.3309E+01
-3.5029E+01 -1.3309E+01
```

The default system is a second-order system. That is, the denominator of the open-loop function is second-order.

Damping Factor

> BASIC ROOT LOCUS DAMPING FACTOR RULE: In a second-order system, or a system with dominant poles (i.e., poles that are significantly closer to the imaginary axis than any other poles) that behave as a second-order system, *the cosine of the angle from the negative real axis to the line extending from the origin (0 + j0) to one dominant complex pole is the damping factor for the system.*

For the system of Figure 12, one of the highlighted poles is at

$$s = -3.5029E{+}01 + j1.3309E{+}01$$

Or, $$s = -35.029 + j13.309$$

Taking the arctangent, $$\tan^{-1}\frac{13.309}{35.029} = 20.804°$$

Taking the cosine of the angle, $$\cos 20.804° = 0.9348$$

So when RootLocus starts with its default coefficients, and the gain is adjusted to 69.957, the damping factor of the default system is 0.9348.

Determining Critical Damping

Consider a second-order system like the one in Figure 12, or a system with dominant poles (i.e., poles that are significantly closer to the imaginary axis than any other poles) that behave as a second-order system: *When the dominant pole-pair just reaches the real axis, called a break-in point, the cosine is 1, so that the damping ratio at that point is the classical critical damping. Therefore, the gain needed to achieve critical damping also can be easily determined using RootLocus.*

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RootLocus Can Determine the Damping Factor for You

RootLocus provides a choice labeled EST. DAMPING FACTOR. Once the root locus has been plotted, you can select Yes to enable the estimation of damping factor. When the scrollbar slider is moved to highlight the migrating poles versus gain, a line appears that connects a dominant pole and the origin (0 + j0).

If you had already used the scrollbar to select a pole of interest and then selected Yes under EST. DAMPING FACTOR, the damping factor will not appear instantly. You must move the scrollbar slightly before the damping factor will appear. Once it is displayed, move the scrollbar back to select the pole you want.

Using RootLocus' default starting values, click Calculate to get the root locus of Figure 11. Now click Yes in EST. DAMPING FACTOR. After clicking Yes, slide the scrollbar until you get the condition shown in Figure 13.

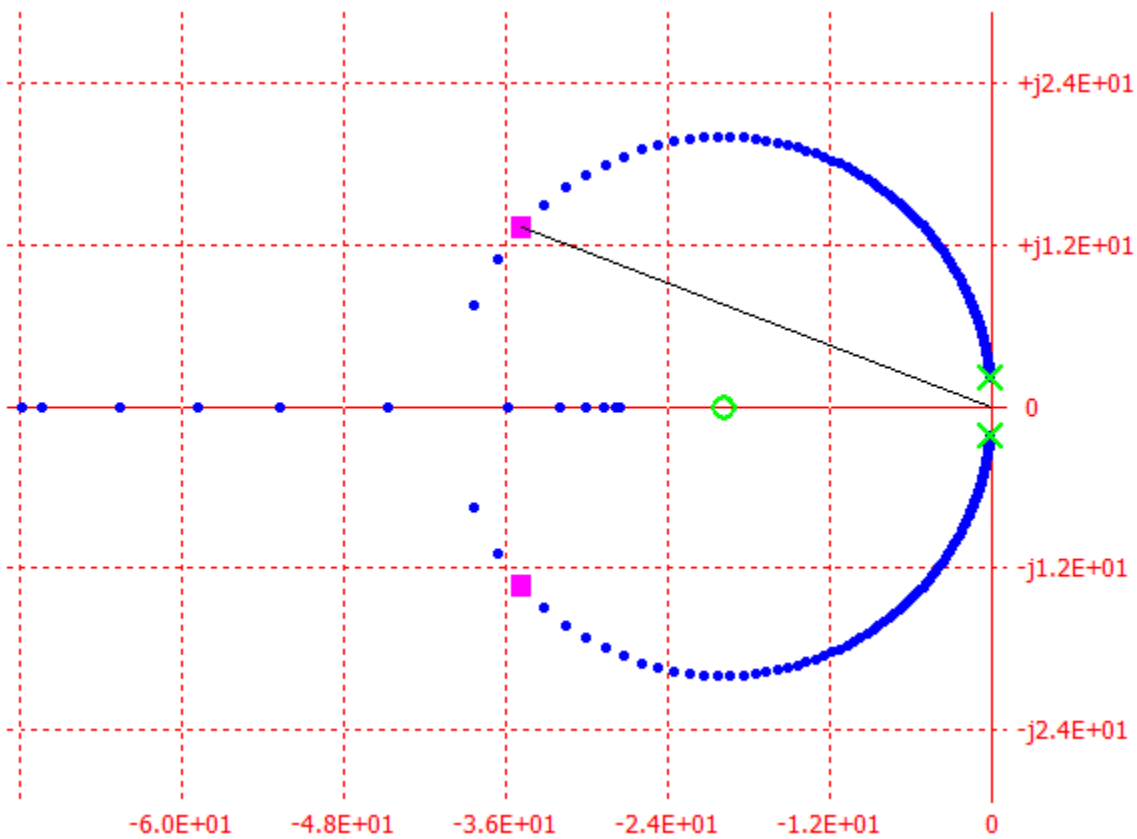

Figure 13. RootLocus can automatically draw a line from the origin to a dominant complex pole

RootLocus calculates the cosine of the angle between the line of Figure 13 and the negative real axis. That angle is illustrated in Figure 14. For the default starting

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conditions and the condition shown in the figure, RootLocus reports these conditions in the right panel:

```
MARKED LOCUS POINTS

GAIN = +6.9957E+01

REAL          IMAG
-3.5029E+01 +1.3309E+01
-3.5029E+01 -1.3309E+01

DAMPING FACTOR = 0.93480
```

Notice that the damping factor has been calculated automatically and that it agrees with the result calculated earlier.

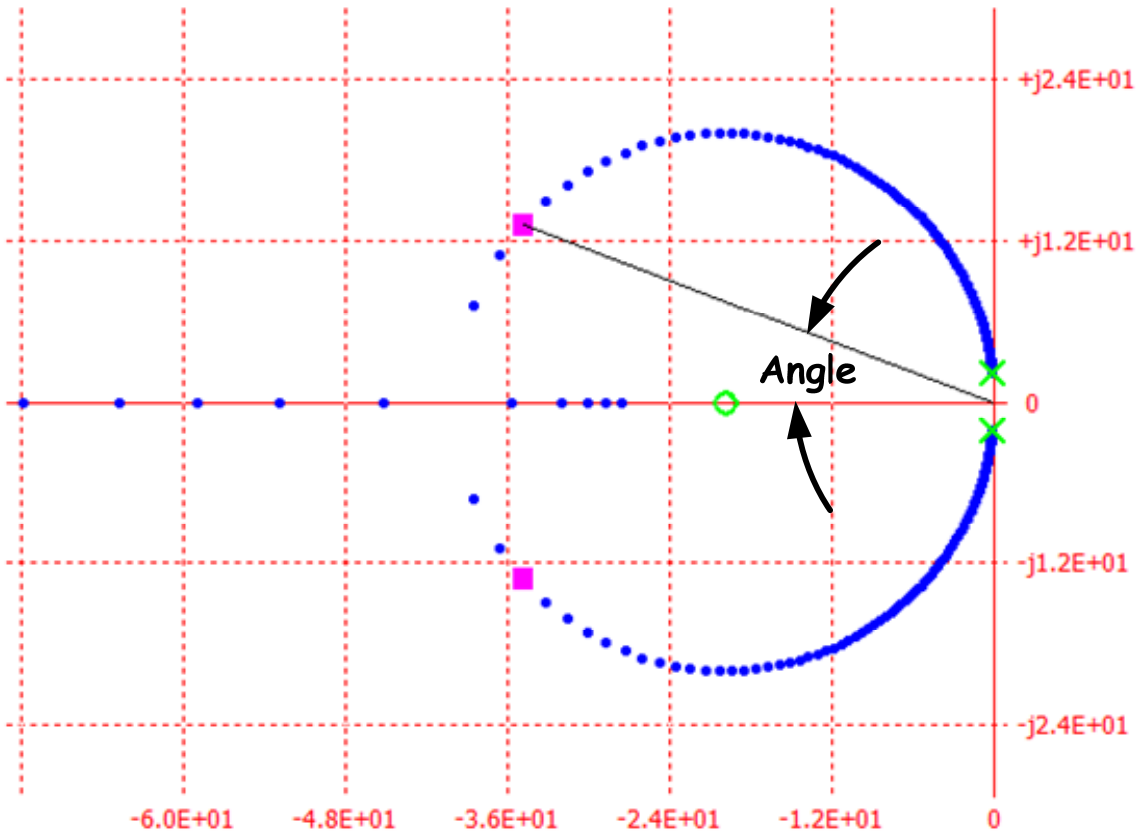

Figure 14. The angle that determines second-order damping

See Page 22 for the Basic Root Locus Damping Factor Rule that is illustrated in Figure 14.

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A Special Note About the "Estimated" Damping Factor

> The term "estimated" is used because the estimated damping factor can be misused. *In general, "damping factor" is only valid for a system that has a second-order denominator.* It is *reasonably* valid when a pair of second-order poles is clearly dominant. Dominant poles are those poles that are significantly closer to the imaginary axis than any other poles, and only when there are no right-half plane poles. When other poles are present and relatively close to the imaginary axis, or poles are in the right-half plane, the estimated damping factor is not valid. To understand and appreciate the concept of dominant poles, refer to any good reference on linear control systems. Do this before depending on the estimated damping factor computed by the RootLocus program.

The RootLocus program will not display the estimated damping factor when it is greater than 1, i.e., when the dominant poles exist on the real axis. Also, RootLocus will not display an estimated damping factor when any poles reside in the right-half plane, a condition of instability.

Keep in mind that the damping factor computed by RootLocus is an estimate because it must be interpreted using a sound understanding of the nature of "damping factor." RootLocus, in its limited way, avoids displaying "damping factor" when it is clear that the estimate would be invalid. However, just because it *does* display "damping factor" does not mean that the displayed damping factor is valid. You must determine if it is valid based upon a sound understanding of damping factor and dominant poles.

RootLocus Estimates Critical Damping

If you are using RootLocus' default starting parameter values, select a PLOTTING DENSITY of Ultra Fine, click the Calculate button again, select Yes in EST. DAMPING FACTOR, then slide the scrollbar slider. You will find that critical damping (i.e., Damping Factor = 1.0) of the default system occurs when the gain is between 79.990 and 80.151, or about 80. Pretty easy!

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Zooming

A zooming feature is included in RootLocus. To expand an area of the plotted root locus, click the mouse somewhere in the plotted area, drag it diagonally in any direction, and unclick the mouse. The plot will then expand to the zoom area that you selected.

To zoom successfully, the zoom area must include any of the plotted information. If you attempt to zoom over an area of the plot that includes no plotted data, RootLocus will tell you.

Only one "zoom" is permitted. This means that once you have zoomed, you cannot zoom again. So, if you find that the zoom area that you have selected isn't exactly what you wanted to see, click Calculate again to start over. Then select a new zoom area.

The amount of zooming is limited. If you try to zoom too much, RootLocus will give you a warning.

Copy the Image to the Clipboard

RootLocus makes documentation easy by providing a simple, convenient way to copy the plot into other documents such as reports.

To copy the plot, simply click the Copy Image button. This puts a copy of the plot into the Windows Clipboard. The copy operation is confirmed by a brief flash of the frame surrounding the image. You can now paste the image into applications that can accept graphic data.

Alternatively, the "C" in the Copy Image button becomes underlined whenever the mouse pointer appears over a plot. This enables a keyboard shortcut. Simply press the 'C' key on the keyboard to copy the image to the Clipboard. The frame around the plot will flash to indicate that the image has been saved to the Clipboard.

Copy the Contents of the Right Panel

If you would like to copy the parameters listed in the right panel, click anywhere in the panel and press Ctrl-A (i.e., press the Ctrl key simultaneously with the 'A' key.) Then press Ctrl-C to copy the contents of the right panel. Finally, click your mouse in the document of your choice where you want the information to appear and press Ctrl-V to paste the information into the document.

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Default Gain

If you were to enter Numerator Coefficients of

10 20 30

and Denominator Coefficients of

5 50 10

they would represent an open-loop function, GH, of the form

$$\frac{10\,s^2 + 20\,s + 30}{5\,s^2 + 50\,s + 10}$$

When the high-order coefficients are factored out, the result is

$$\frac{10}{5}\,\frac{s^2 + 2\,s + 3}{s^2 + 10\,s + 2} = 2\,\frac{s^2 + 2\,s + 3}{s^2 + 10\,s + 2}$$

When the Calculate button is clicked, RootLocus performs these calculations and reports the factor 2 as 2.0000000 in the DEFAULT GAIN window. However, the original numerator and denominator coefficients continue to be displayed

The DEFAULT GAIN is the "gain" of the given system when the numerator and denominator polynomial are reduced to monic form, i.e., when the high-order coefficients are factored out so that both high-order coefficients become 1.0.

Once the scrollbar has been moved from its initial position, the highlighted poles appear on the plot and the Use Default button is enabled. Clicking the Use Default button causes the default gain (2 in this example) to be selected so that the highlighted poles are moved to the locus positions associated with this gain.

The DEFAULT GAIN feature permits you to view the pole positions of the given system before the gain is varied. That is, it allows you to determine if a particular system is stable—or not—before you attempt to make any changes to it.

The default gain can only be positive. However, when the gain range is negative, and the default gain is contained within the range of the absolute values of the Minimum Gain and Maximum Gain, the negative value of the default gain is taken as the default gain.

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Whenever the default gain is selected, either by clicking the Use Default button or by sliding the scrollbar, the label of the Use Default button will be red to identify the condition.

PLOTTING THE STEP AND FREQUENCY RESPONSES

Step Response

RootLocus can plot the step response of the closed-loop system for the selected gain and the calculated pole positions.

If RootLocus is already open, close it. Now open RootLocus to recover the default starting parameter values. Using the default conditions, click on the Calculate button. Now click Yes in EST. DAMPING FACTOR. After clicking Yes, slide the scrollbar until you get the condition shown in Figure 14.

Now click on the Step Response button. The plot should look like Figure 15.

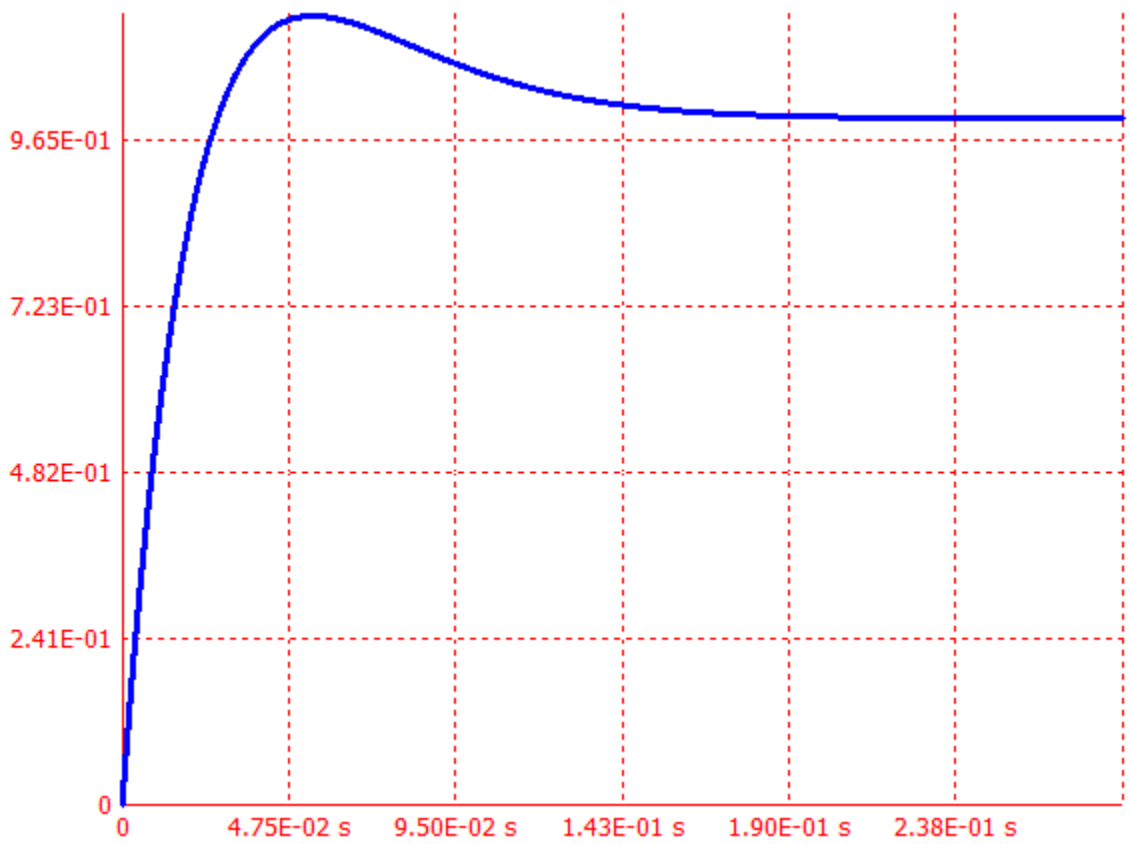

Figure 15. Step response for the conditions in Figure 14

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The plotted step response is based on the closed-loop transfer function

$$\frac{C(s)}{R(s)} = \frac{K\,G(s)}{1 + K\,G(s)H(s)}$$

and an input step with an amplitude of 1.0.

Notice that the caption on the Step Response button is now red. Many of the other functions are disabled, which is indicated by each affected function being dimmed. Those functions that are not dimmed are still usable.

When the mouse pointer is placed over the red Step Response button, the hint reads

CLICK AGAIN TO RETURN TO NORMAL MODE

To return to "normal" mode, click the red Step Response button again. After clicking the button, Step Response returns to black and all disabled functions are re-enabled.

If the Step Response button is not red, click it again so that Step Response is red. Notice that when the mouse pointer is placed over the step response plot that the hint reads

STEP RESPONSE: Maximum = 1.1449, Minimum = 0.0000, Initial = 0.0000, Final = 0.99640

The MAXIMUM value is the highest calculated value of the plot *inside the plot boundaries*.

The MINIMUM value is the lowest calculated value of the plot *inside the plot boundaries*.

The INITIAL value is the value of the plot *at the left edge of the plot*.

The FINAL value is the value of the plot *at the right edge of the plot*.

Frequency Response

RootLocus can plot the frequency response of the closed-loop system for the selected gan and the calculated pole positions.

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If RootLocus is already open, close it. Now open RootLocus to recover the default starting parameter values. Using the default conditions, click on the Calculate button. Now click Yes in EST. DAMPING FACTOR. After clicking Yes, slide the scrollbar until you get the condition shown in Figure 14.

Now click on the Freq Response button. The plot should look like Figure 16.

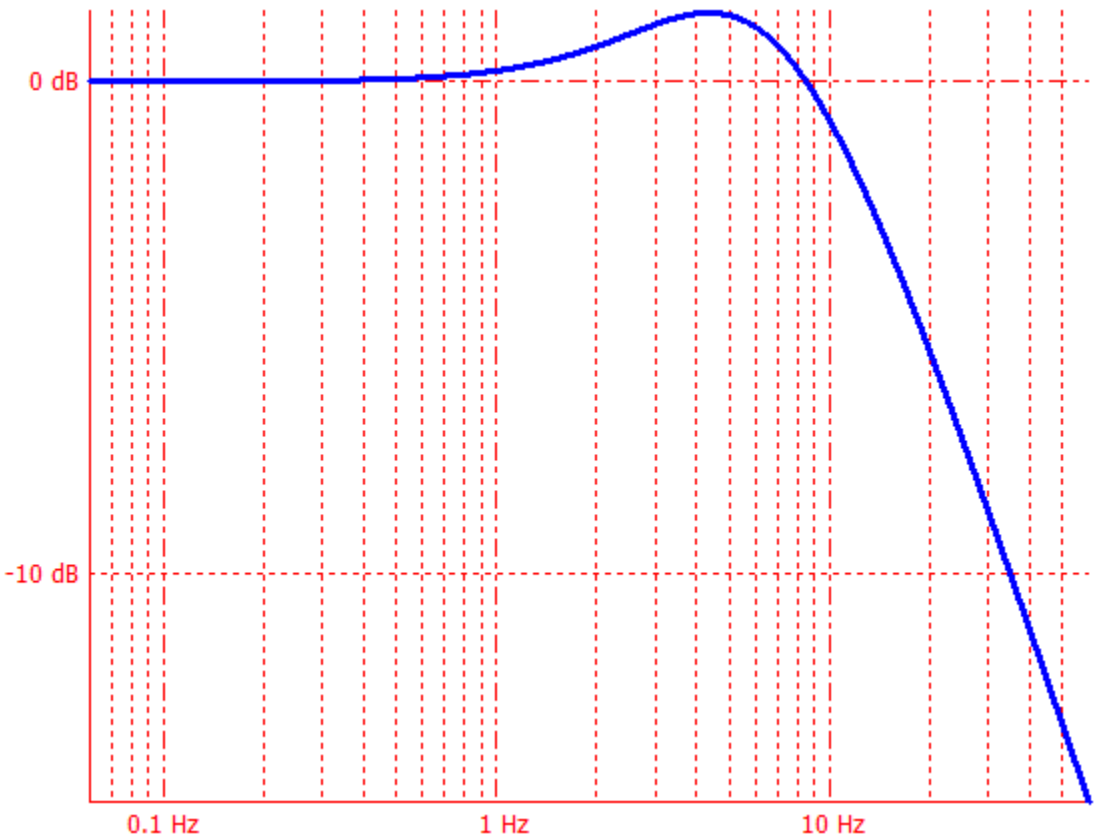

Figure 16. Frequency response for the conditions in Figure 14

The plotted frequency response is based on the closed-loop transfer function

$$\frac{C(s)}{R(s)} = \frac{K\,G(s)}{1 + K\,G(s)H(s)}$$

and an input signal of amplitude 1.0.

Notice that the caption on the Freq Response button is now red. Many of the other functions are disabled, which is indicated by each affected function being dimmed. Those functions that are not dimmed are still usable.

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When the mouse pointer is placed over the red Freq Response button, the hint reads

CLICK AGAIN TO RETURN TO NORMAL MODE

To return to "normal" mode, click the Freq Response button again. After clicking the button, Freq Response returns to black and all disabled functions are re-enabled.

If the Freq Response button is not red, click it again so that Freq Response is red. Notice that when the mouse pointer is placed over the frequency response plot that the hint reads

FREQUENCY RESPONSE: Maximum = +1.3665 dB, Minimum = -14.630 dB, Initial = -0.030110 dB, Final = -14.630 dB

The MAXIMUM value is the highest calculated value of the plot *inside the plot boundaries*.

The MINIMUM value is the lowest calculated value of the plot *inside the plot boundaries*.

The INITIAL value is the value of the plot *at the left edge of the plot*.

The FINAL value is the value of the plot *at the right edge of the plot*.

<u>The Frequency Response Plot is Valid Only Under Certain Conditions</u>

> *The calculated frequency response is meaningful only if the system is stable. <u>It is not meaningful if the system is unstable.</u> Therefore, whenever any pole exists on the jω axis or in the right-half plane, the plotted frequency response is not valid.*

<u>Saving the Step and Frequency Response Data</u>

The *File* menu contains an item called *Save Step, Frequency, and Bode Response Data*. When this feature is selected, a check mark will appear next to the item.

Saving Step Response Data

When *File > Save Step, Frequency, and Bode Response Data* is enabled and the Step Response button is clicked, the step response data is saved to a default file

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named STEP_RESPONSE.TXT. This file will be placed in the folder where ROOTLOCUS.EXE is located.

The data in STEP_RESPONSE.TXT is the data used to plot the step response. The file comprises three tab-separated columns: time increment in seconds, amplitude, step number.

The data in STEP_RESPONSE.TXT can be pasted into a Microsoft Excel® spreadsheet. To do this, open STEP_RESPONSE.TXT using Windows Notepad. Click inside the file listing and press Ctrl-A to select all of the data, then press Ctrl-C to copy all of the data. Open Excel and select the upper-left cell where you want the data to be located. Then press Ctrl-V to paste the data into Excel.

One thing you can do with the data once it's in Excel is create a plot. Use the first data column as the horizontal (time) axis of the plot. Use the second data column as the vertical (amplitude) axis of the plot. The third data column simply provides a reference count that you can use if needed.

Saving Frequency Response Data

When *File > Save Step, Frequency, and Bode Response Data* is enabled and the Freq Response button is clicked, the frequency response data is saved to a default file named FREQ_RESPONSE.TXT. This file will be placed in the folder where ROOTLOCUS.EXE is located.

The data in FREQ_RESPONSE.TXT is the data used to plot the frequency response. The file comprises three tab-separated columns: sample frequency in hertz, amplitude in decibels, step number.

The data in FREQ_RESPONSE.TXT can be pasted into a Microsoft Excel® spreadsheet. To do this, open FREQ_RESPONSE.TXT using Windows Notepad. Click inside the file listing and press Ctrl-A to select all of the data, then press Ctrl-C to copy all of the data. Open Excel and select the upper-left cell where you want the data to be located. Then press Ctrl-V to paste the data into Excel.

One thing you can do with the data once it's in Excel is create a plot. Use the *second* data column as the vertical (decibel) axis of the plot. Use the *third* data column as the horizontal (frequency) axis of the plot. This method will permit the frequency response to be plotted along a logarithmic horizontal axis since each sample point was calculated using logarithmic frequency scaling.

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The first data column is the list of frequency sample values. Using this data as the horizontal (frequency) axis in Excel causes the plot to be distorted. Use the third column instead to create an undistorted plot.

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BODE PLOTS

RootLocus can plot the Bode responses for a system's *open-loop* poles and zeros.

> **It is important to remember that a root locus plots the loci of the *closed-loop* poles. A Bode plot, on the other hand, plots the amplitude response and phase response of the *open-loop* poles and zeros.**

To repeat an earlier statement, the Bode plot method analyzes a linear control system for a single value of gain. A control loop's gain margin and phase margin reveal if the loop is stable *for that gain only!* But because engineers tend to be comfortable with the Bode method, RootLocus has the ability to plot a system's Bode plot.

<u>The Mechanics of a Bode Plot</u>

Suppose Eq. 20 is the transfer function of a system that you want to analyze.

$$\frac{s-1}{s^2+2s+2} = \frac{s-1}{(s+1-j)(s+1+j)} \qquad \text{Eq. 20}$$

Figure 17 is the pole-zero diagram for Eq. 20.

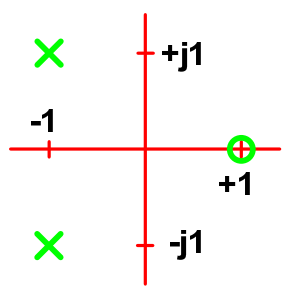

Figure 17. Bode example pole-zero diagram

Here is something you should realize about pole-zero diagrams:

> **Of all the space on the complex plane, you can operate only on the positive imaginary axis.**

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This fact might sound wrong at first but consider that, given a transfer function, you can substitute only jω for s. That is, you can apply DC (i.e., j0 = 0) or some nonzero frequency (i.e., an arbitrary jω), but you *cannot* apply a negative frequency or anything else that is not on the positive imaginary axis. In other words, you can't generate –ω or ±jω ± σ, so it makes no sense to attempt to analyze the response of a transfer function using those terms.

So when you substitute some jω for s in a transfer function like Eq. 20, it's the equivalent of simply selecting a point on the positive imaginary axis of Figure 17.

Bode Amplitude Response

When a sinusoidal signal passes through the transfer function represented by Eq. 20, the transfer function modifies both the amplitude and phase of the signal.

The amplitude response of the transfer function can be determined by taking the magnitude of each term. That is, Eq. 20 becomes Eq. 21.

$$\frac{|s-1|}{|s+1-j||s+1+j|} \qquad \text{Eq. 21}$$

Substituting using s = jω,

$$\frac{|j\omega-1|}{|j\omega+1-j||j\omega+1+j|} \qquad \text{Eq. 22}$$

Rearranging by grouping the real and imaginary parts of each term,

$$\frac{|-1+j\omega|}{|1+j(\omega-1)||1+j(\omega+1)|} \qquad \text{Eq. 23}$$

Each term in Eq. 23 is simply the vector distance to jω from the zero and the poles of Figure 17. The vectors are illustrated in Figure 18.

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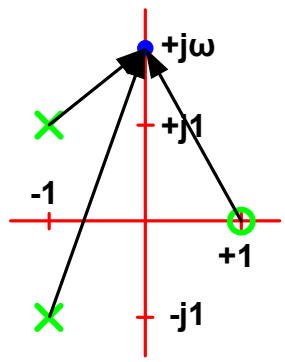

Figure 18. Bode magnitudes

The amplitude response of the transfer function illustrated in Figure 18 can be estimated by observation based on Eq. 23. For example, for the position of the marked jω in the figure, the individual lengths are $L_Z \approx 2.1$, $L_{P1} \approx 1.4$, and $L_{P2} \approx 3.1$ so that $L_{TOTAL} \approx 2.1 / [(1.4)(3.1)] = 0.48$. And 20 log 0.48 = -6.3 dB.

Of course, any open-loop gain in the system (i.e., K of the root locus) simply multiplies the amplitude response of Eq. 23 or the graphic results of Figure 18.

Bode Phase Response

The phase response of the transfer function can be determined by taking the angle of each term using the arctangent function. That is, adapting Eq. 23 gives Eq. 24.

$$\frac{\angle(-1+j\omega)}{\angle[1+j(\omega-1)]+\angle[1+j(\omega+1)]} = \angle(-1+j\omega)-\angle[1+j(\omega-1)]-\angle[1+j(\omega+1)] \quad \text{Eq. 24}$$

Eq. 24 shows that the total angle due to the poles is subtracted from the total angle due to the zeros. Care must be taken to ensure that the angles calculated using this method are represented in the correct quadrant. The angles for Eq. 24 are illustrated in Figure 19.

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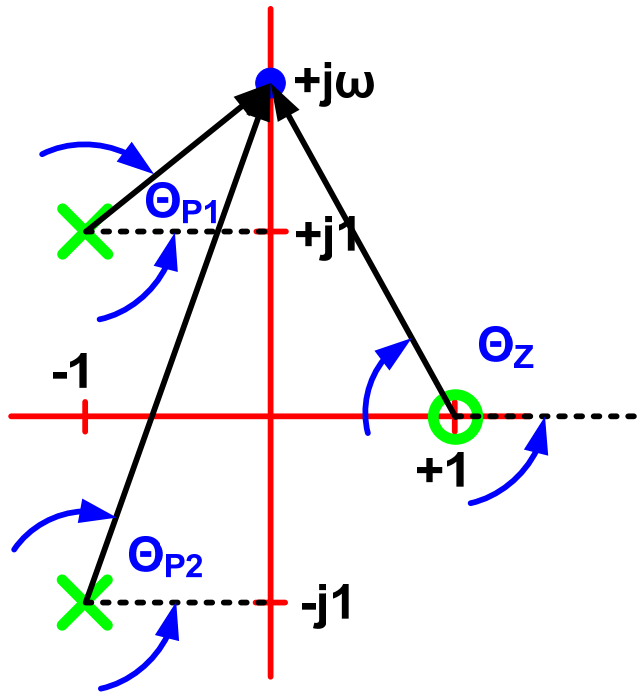

Figure 19. Bode phases

The beauty of the graphical approach illustrated in Figure 19 is that the total phase angle can be estimated by observation based on Eq. 24. For example, given the position of the marked jω in the figure, $\theta_Z \approx 120°$, $\theta_{P1} \approx 45°$, and $\theta_{P2} \approx 70°$. So the total angle for the example system is approximately $\theta_{TOTAL} \approx 120° - 45° - 70° = 5°$. Simple!

A closed-loop system typically uses negative feedback. This inversion adds another 180° of phase shift. So, if the phase just estimated were the open-loop phase response of a system with negative feedback, the total estimated phase would be 180° + 5° = 185°. If positive feedback were used, then the total estimated phase shift would be the original 5°.

<u>RootLocus Calculates the Bode Response</u>

Start RootLocus and enter the coefficients from Eq. 20 using these values:

Numerator coefficients:	1 -1
Denominator coefficients:	1 2 2
Maximum gain:	1
Minimum gain:	0
Plotting density:	Ultra Fine

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Click the Calculate button. Then slide the scrollbar to get the maximum gain condition shown in Figure 20.

Now click the Bode Plot button to get the Bode plot of Figure 21. The Bode Plot button is now red. Some other functions are disabled as indicated by those functions being dimmed. To return to normal mode, click the red Bode Plot button again.

The Bode Plot button is not enabled unless the scrollbar slider is first moved away from its starting point and the gain is not zero. (There can be no feedback—and no instability—if the gain is zero.)

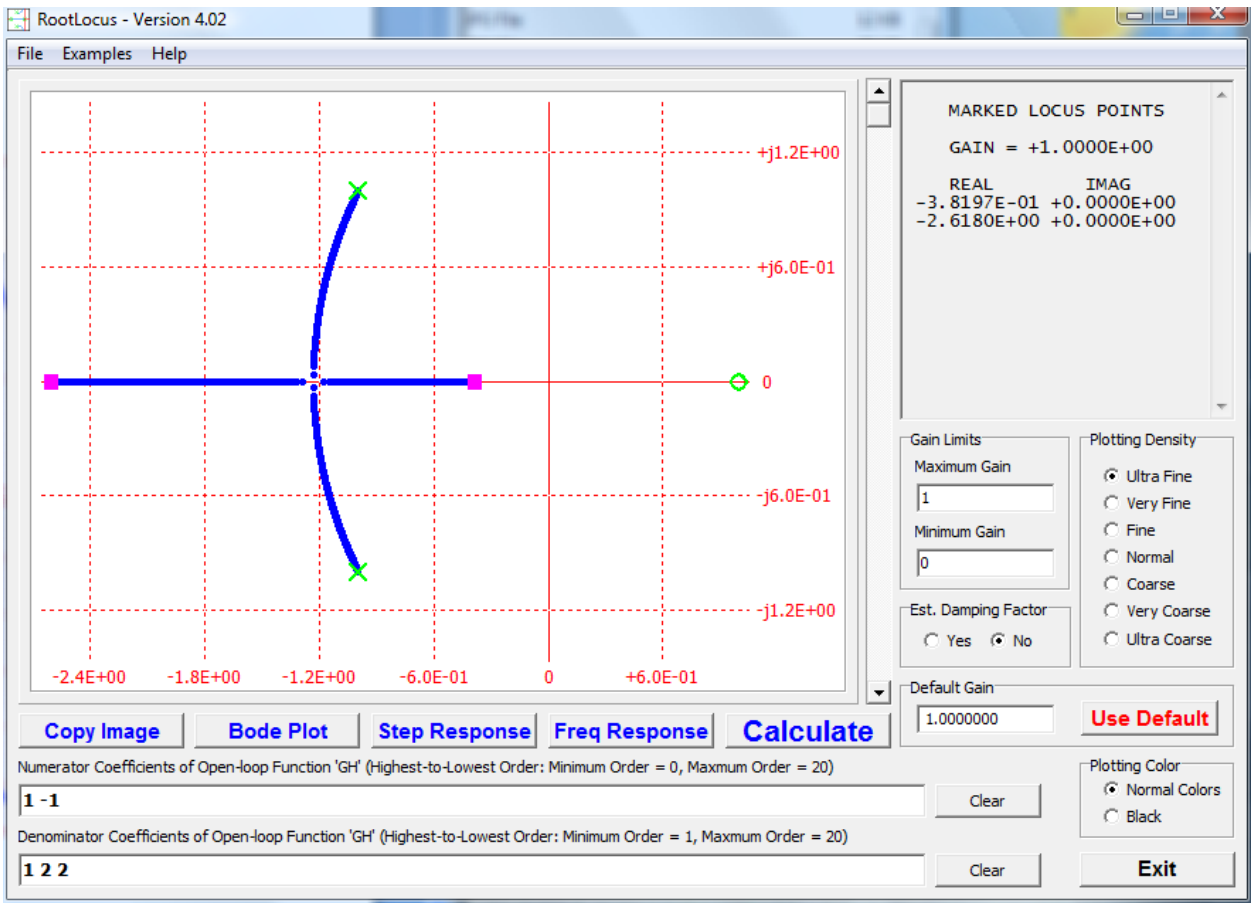

Figure 20. Root locus for the Bode plot example

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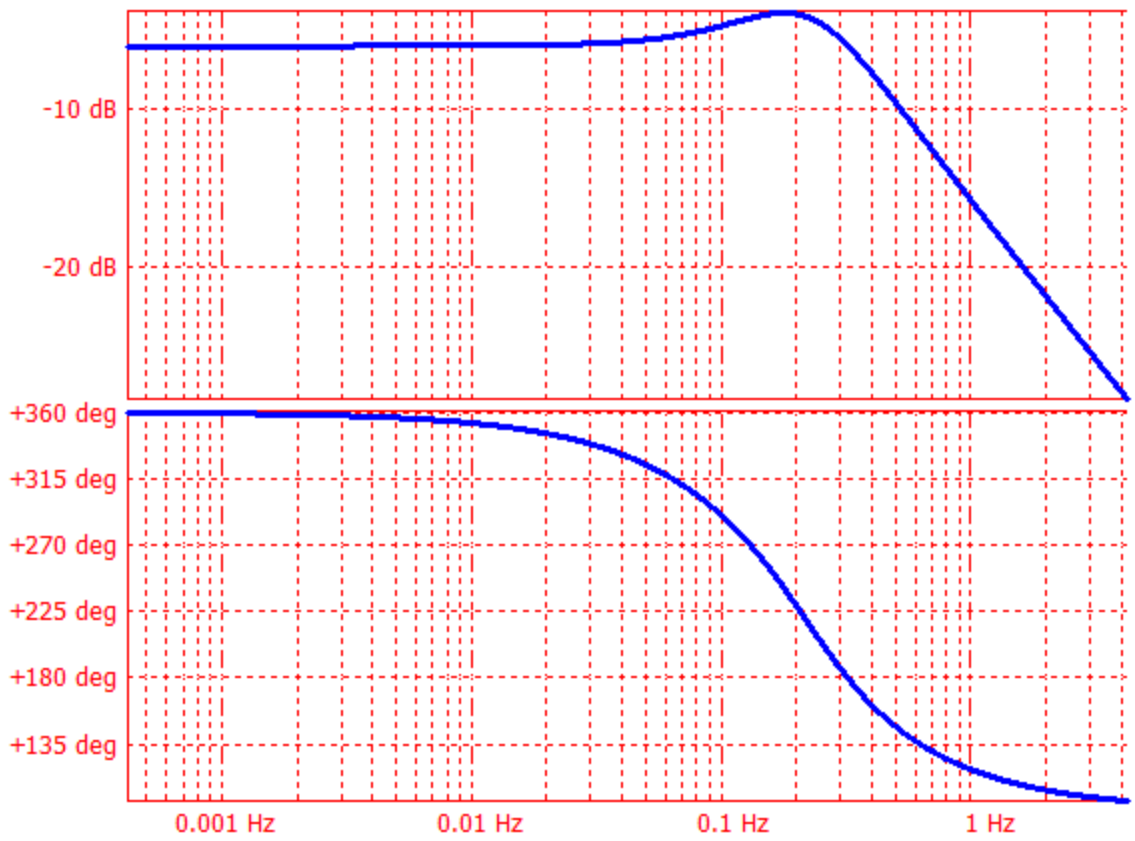

Figure 21. Bode plot for the case shown in Figure 20

Before proceeding, be sure that the Bode Plot button caption is red. If it isn't red, click the button.

The marked jω position in Figure 18 and Figure 19 is roughly jω = j2. Therefore, $f = \omega / 2\pi \approx 2 / 2\pi = 0.318$ Hz.

When you place the mouse pointer over the Bode plot, a vertical cursor line appears. When the line is present, the parameters associated with the line appear in the panel at the right.

Notice the highlighted box immediately above. Placing the mouse pointer over the Bode plot causes a cursor line to appear.

Place the mouse pointer over the plot and move it horizontally until the frequency in the panel at the right is as close to 0.318 Hz as possible. The best you can do is 0.3205 Hz. The results that you obtain should look like Figure 22.

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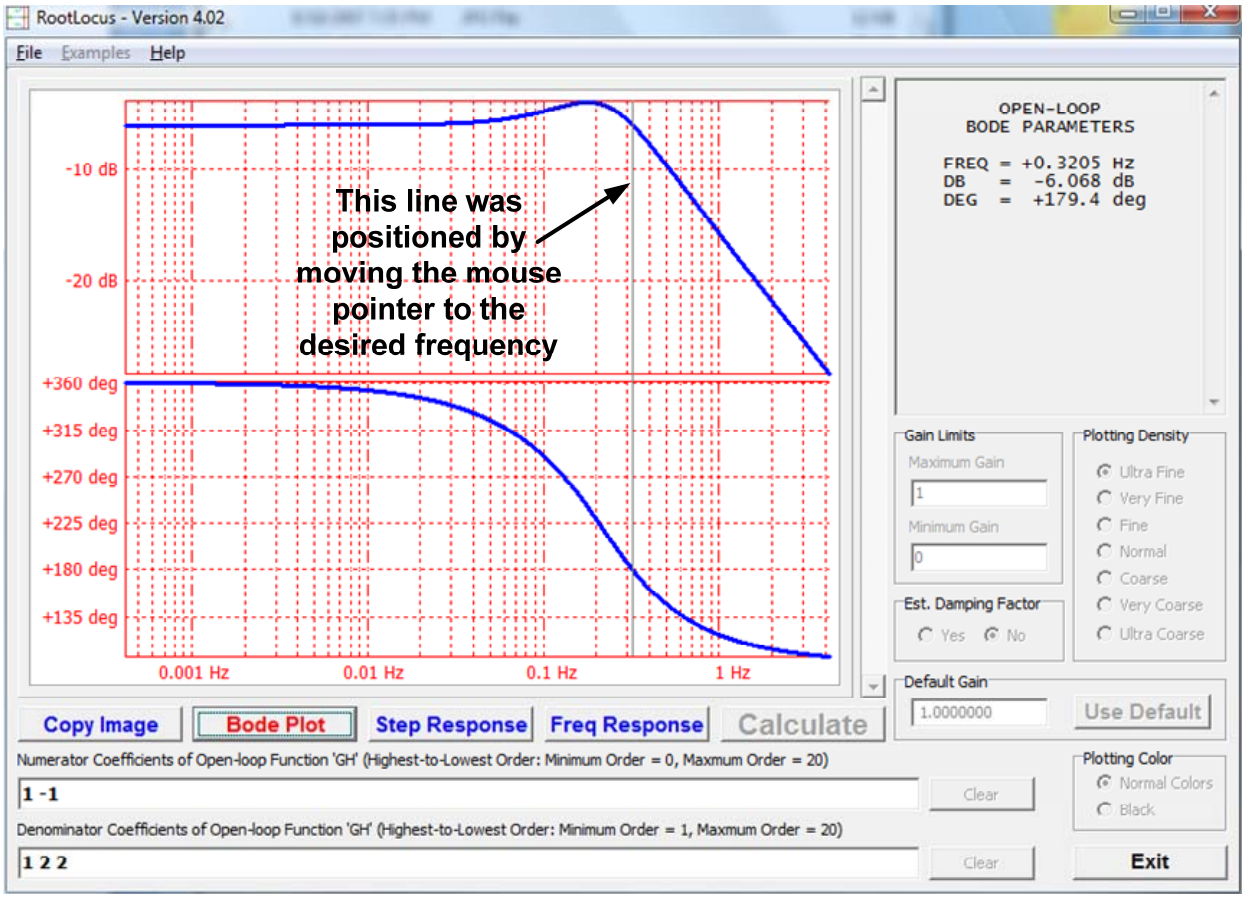

Figure 22. Bode plot example with the cursor positioned by the mouse

RootLocus calculates about 2000 points when it computes the Bode plot. But the displayed plot doesn't have this much resolution. Therefore, the display's resolution is coarser than you might like. This explains why the closest you can get to 0.318 Hz using the cursor is 0.3205 Hz when moving the mouse pointer over the Bode plot.

When the mouse pointer is over the Bode plot and the vertical cursor line is visible, clicking the mouse button copies the contents of the right panel to the Windows Clipboard. This action is confirmed by a brief flash of the panel.

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Estimated Response versus Calculated Response

How do the results obtained in Figure 22 compare with the Bode plot estimates that were made on Pages 36 and 37? Those estimates were -6.3 dB and 185°, respectively. The true numbers from Figure 22 are -6.068 dB and 179.4°. So the estimates based on visualizing the graphics plot were reasonably close.

The Effect on the Bode Plot of Adjusting the Scrollbar

Since the scrollbar in Figure 20 had been set to the maximum gain and Maximum Gain = 1, the multiplier on the transfer function was 1. This is the reason that -6.068 dB was obtained in Figure 22 so as to agree with the earlier estimates. *When the scrollbar is moved to increase or decrease the gain, the Bode dB response curve moves up or down, respectively. The phase response is not affected by the position of the scrollbar slider.*

Negative Feedback Adds Another 180°

Since the gain range used in Figure 22 is *positive*, *negative* feedback is represented. (See Page 3 for a brief description of gain range and feedback polarity.) Negative feedback corresponds to an inversion that requires an extra 180° to be added to the phase response of the transfer function. This was done to get the phase results in the figure.

You might want to change the gain range to Maximum Gain = 0 and Minimum Gain = -1 to see the effects of using positive feedback. If you try this, you will find that when the cursor is placed as close as possible to 0.318 Hz, the phase is about 0° instead of the approximately 180° found above.

Gain Margin and Phase Margin

A Bode plot reveals system stability by using parameters called the gain margin and phase margin. See Page 63 for an example that explains how to use the Bode Plot function in RootLocus to determine gain margin and phase margin.

<u>Saving the Bode Plot Data</u>

The *File* menu contains an item called *Save Step, Frequency, and Bode Response Data*. When this feature is selected, a check mark will appear next to the item.

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When *File > Save Step, Frequency, and Bode Response Data* is enabled and the Bode Plot button is clicked, the Bode plot data is saved to a default file named BODE_RESPONSE.TXT. This file will be placed in the folder where ROOTLOCUS.EXE is located.

The data in BODE_RESPONSE.TXT is the data used to plot the Bode responses. The file comprises four tab-separated columns: sample frequency in hertz, amplitude in decibels, phase in degrees, step number.

The data in BODE_RESPONSE.TXT can be pasted into a Microsoft Excel® spreadsheet. To do this, open BODE_RESPONSE.TXT using Windows Notepad. Click inside the file listing and press Ctrl-A to select all of the data, then press Ctrl-C to copy all of the data. Open Excel and select the upper-left cell where you want the data to be located. Then press Ctrl-V to paste the data into Excel.

One thing you can do with the data once it's in Excel is create Bode amplitude and phase response plots. Use the *second* data column as the vertical (decibel) axis of the amplitude response plot. Use the *third* column as the vertical (degree) axis of the phase response plot. Use the *fourth* data column as the horizontal (frequency) axis for each of the plots. This method will permit the responses to be plotted using a logarithmic horizontal (frequency) axis since each sample point was calculated using logarithmic frequency scaling.

The first data column is the list of frequency sample values. Using this data as the horizontal (frequency) axis in Excel will cause the plots to be distorted. Use the fourth column instead to create an undistorted plot.

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MORE EXAMPLES USING RootLocus

EXAMPLE 1: More Parameters Available from the Root Locus

Open RootLocus and enter these values:

Numerator coefficients:	1
Denominator coefficients:	1 10 5
Maximum gain:	100
Minimum gain:	0
Plotting density:	Normal

RootLocus lets you load all of the values listed above by selecting *Examples > Example 1* from the menu. This choice also computes the example root locus automatically.

Click the Calculate button. RootLocus should look like Figure 23. This is a simple second-order system. The migrating poles will continue along a vertical line to $\pm j\infty$. *Since the entire locus is contained in the left-half plane, this system is unconditionally stable.*

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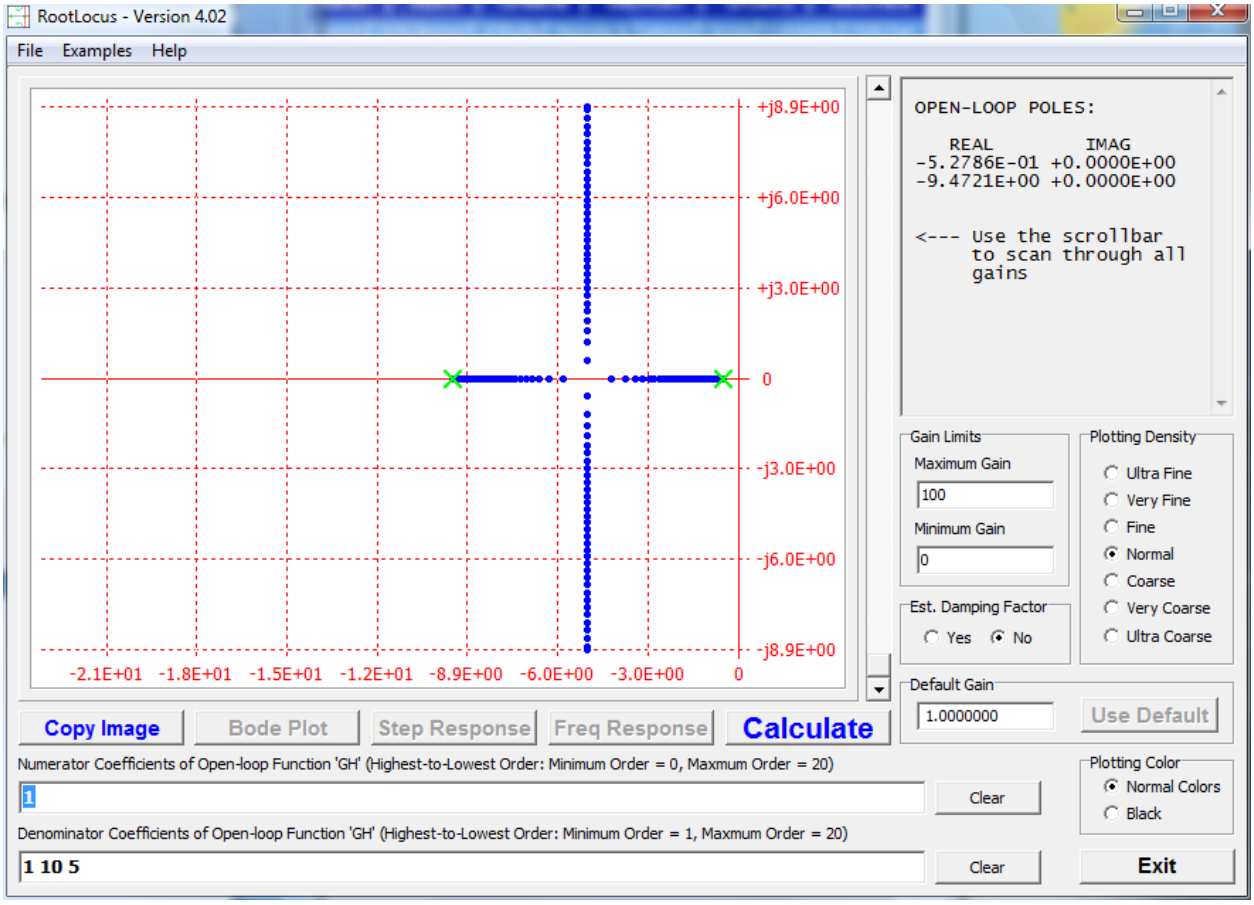

Figure 23. Example 1 after clicking Calculate

Now click Yes in the EST. DAMPING FACTOR group. Then slide the scrollbar until these values appear in the right panel:

```
MARKED LOCUS POINTS

GAIN = +6.3414E+01

REAL          IMAG
-5.0000E+00 +6.5889E+00
-5.0000E+00 -6.5889E+00

DAMPING FACTOR = 0.60450
```

Your display should look like Figure 24. First notice that the point where the locus becomes complex is called a breakaway point. Then notice the other labels.

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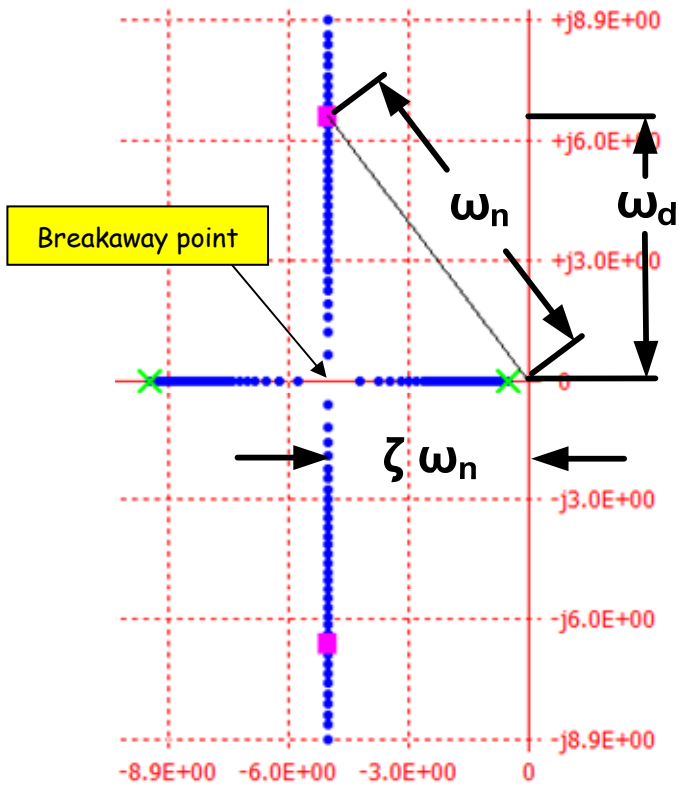

Figure 24. Example 1 with new information

The *damped natural frequency*, ω_d, is the distance from the negative real axis to the complex pole of interest. It can be read from the right panel as ω = 6.5889, or f = ω / 2π = 1.0487 Hz. Since the poles are in the left-half plane, the system will behave with damped sinusoidal ringing. The ringing frequency at this gain is 1.0487 Hz. Clearly, as gain increases, this system's damped frequency increases.

The system's *undamped natural frequency*, ω_n, is the length of the line from the origin to the pole of interest. The system's closed-loop response has the form

$$\frac{C(s)}{R(s)} = \frac{a}{s^2 + 2\varsigma\,\omega_n + \omega_n^2}$$

The undamped natural frequency, ω_n, in the equation is the length of the line shown in Figure 24. Variable ζ (zeta) is the damping factor. Based on the information in the right panel,

$$\omega_n = \sqrt{\omega_d^2 + (\varsigma\,\omega_n)^2} = \sqrt{\omega_d^2 + \sigma^2} = \sqrt{6.5889^2 + 5.0000^2} = 8.2713$$

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Note that $\zeta\omega_n = \sigma$, the real part of the complex pole. The right panel tells you that the damping factor is $\zeta = 0.60450$. (Since the system in this example is a true second-order system, the "estimated" damping factor is the true damping factor.) The damping factor is the cosine of the angle between the line representing ω_n and the negative real axis, so it makes sense that the distance from the imaginary axis to the complex poles is $\zeta\omega_n$. Check that fact by multiplying the listed damping factor, 0.60450, by the computed ω_n to get 0.60450 x 8.2713 = 5.0000. There it is!

Finally, consider a little more mathematical look at the system in this example. When the closed-loop system is driven by a step function, and $\zeta < 1$, it has a time response of the following form.

$$c(t) = a_0 + a_1\, e^{-\varsigma\omega_n t}\, \sin\left(\omega_d\, t + \phi\right)$$

Compare the variables in this equation with the variables listed in Figure 24. All of the key variables are available from the root locus. It should be clearer now that, as the poles move farther to the left of the imaginary ($j\omega$) axis (i.e., as the term $-\zeta\omega_n$ takes on a larger magnitude,) the ringing decays faster. Conversely, as the poles move closer to the imaginary axis, the decay becomes slower.

When the roots lie on the $j\omega$ axis, the exponent, $-\zeta\omega_n t$, is zero (since $\zeta\omega_n$ is zero) and the ringing goes on forever.

When the poles move to the right of the imaginary axis, the exponent changes sign and becomes $+\zeta\omega_n t$. Therefore, c(t) increases without bound when poles reside in the right-half plane. The system is clearly unstable when poles are in the right-half plane.

Looking at the equation for c(t) once more, the ringing frequency is clearly ω_d since $\omega_d t$ is an argument of the sine function.

Using the results found by RootLocus for this example, the function can be rewritten as

$$c(t) = a_0 + a_1\, e^{-5.0000\, t}\, \sin\left(6.5889\, t + \phi\right)$$

The *time constant* is $1 / \zeta\omega_n = 1 / |\sigma|$. So the time constant of this system is 1 / 5 = 0.2 seconds. The smaller the time constant, the faster the decay. "Complete" decay takes roughly 5 time constants, about 5 x 0.2 seconds = 1 second in this case.

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To see what the step response of the Example 1 system is, click the Step Response button. The step response is shown in Figure 25.

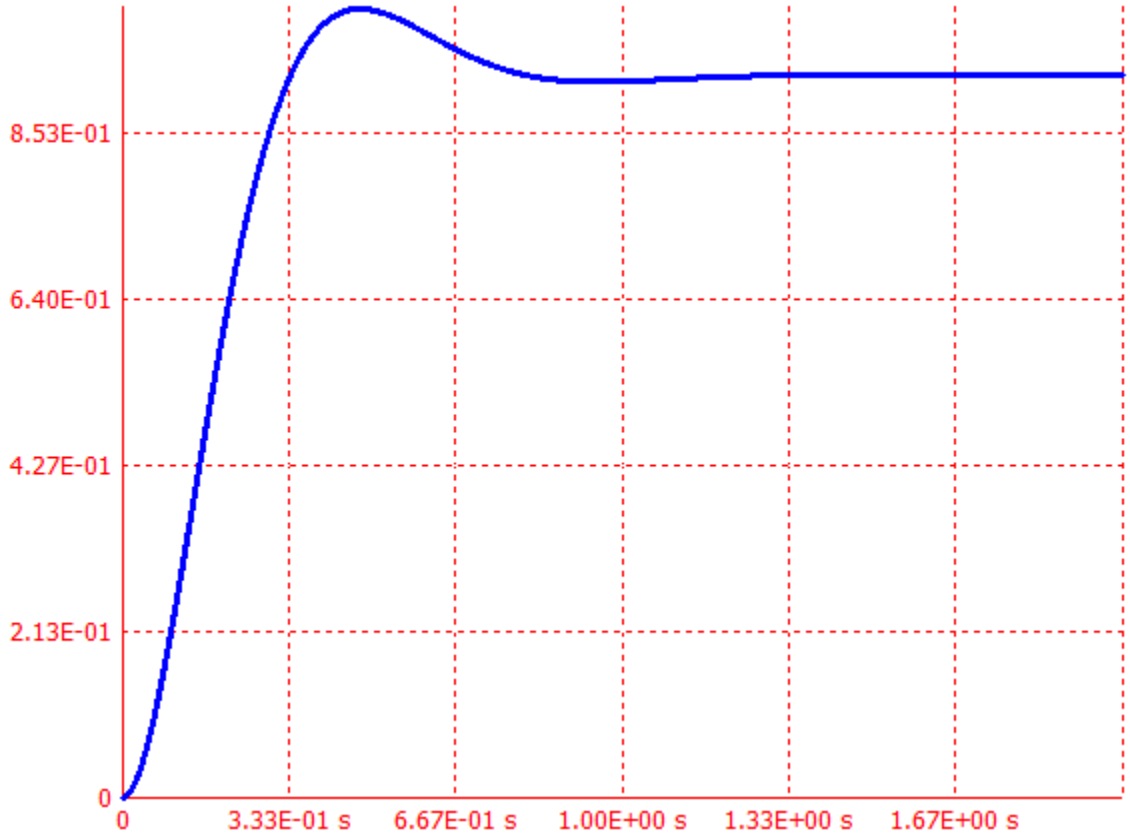

Figure 25. Step response of Example 1

Clicking the Freq Response button provides the frequency response shown in Figure 26.

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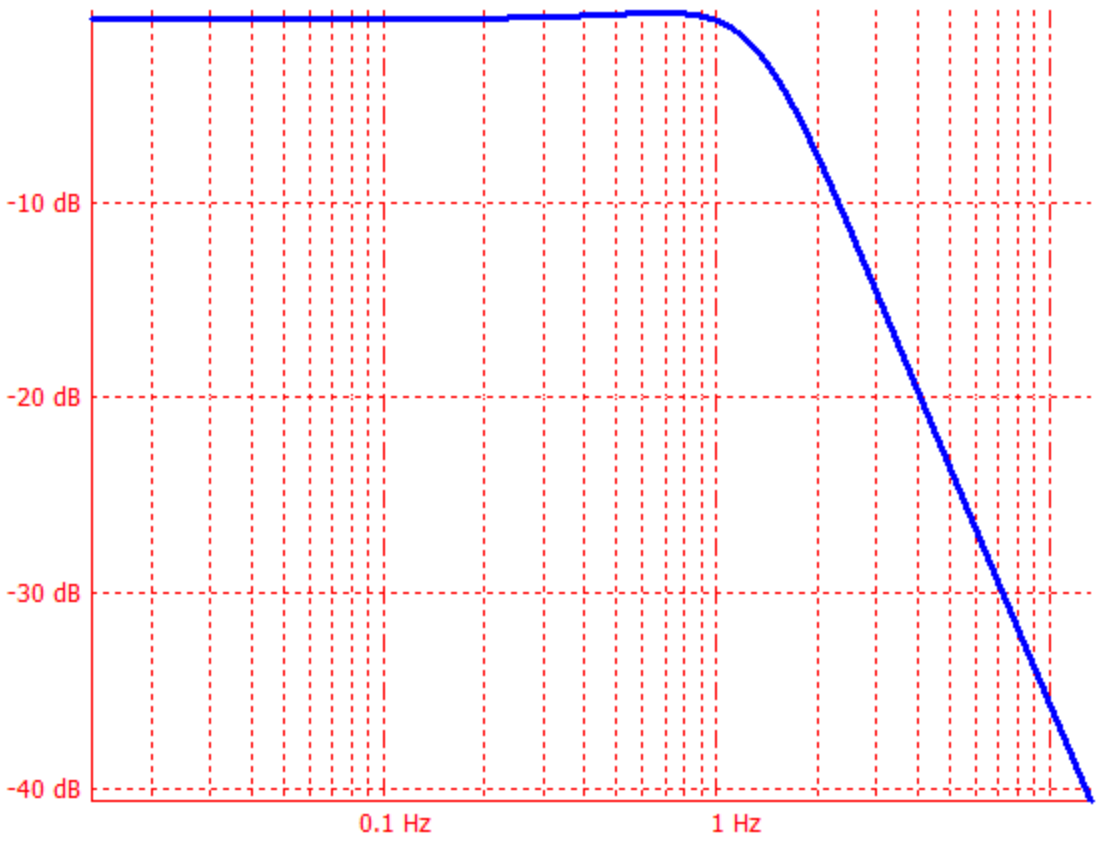

Figure 26. Frequency response of Example 1

Click the red Freq Response button again to return to normal mode

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EXAMPLE 2: Ranges of Stability

Suppose you want to examine the stability of a system that has this open-loop transfer function:

$$G(s)H(s)=\frac{s^2+2s+3.89}{s(s+6.6)(s+4.3)(s^2+1.6s+0.8)}$$

First, expand the denominator into a simple polynomial to get the transfer function into this form:

$$G(s)H(s)=\frac{s^2+2s+3.89}{s^5+12.5s^4+42.62s^3+54.128s^2+22.704s}$$

Open RootLocus and enter these values:

Numerator coefficients:	1 2 3.89
Denominator coefficients:	1 12.5 46.62 54.128 22.704 0
Maximum gain:	300
Minimum gain:	0
Plotting density:	Ultra Fine

Notice the "0" at the end of the list of denominator coefficients. It's easy to forget to include this term.

RootLocus lets you load all of the values listed above by selecting *Examples > Example 2* from the menu. This choice also computes the example root locus automatically.

Click the Calculate button. RootLocus should look like Figure 27. This is a fifth-order system. Some regions of the locus fall in the right-half plane. RootLocus lets you easily identify the gain ranges for which it is stable or unstable.

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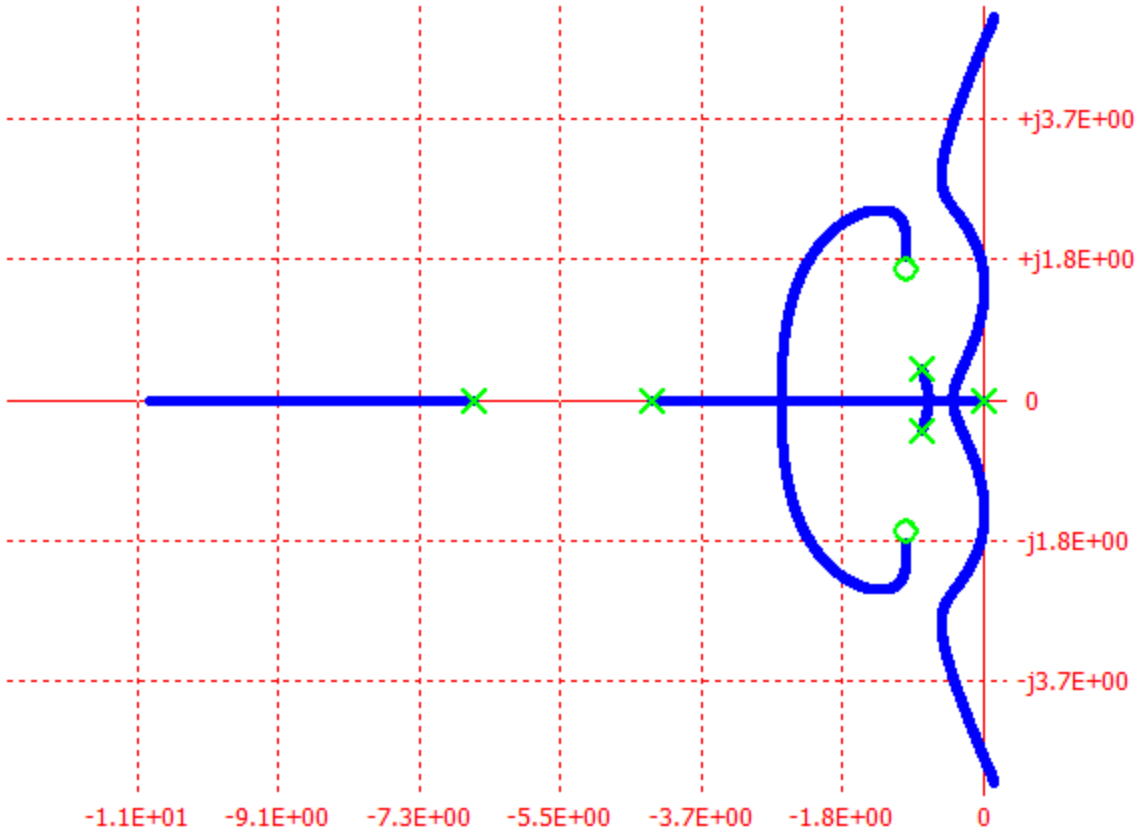

Figure 27. Example 2 after clicking Calculate

To look for the places where the locus moves into the right-half plane, slide the scrollbar slider and watch the list of MARKED LOCUS POINTS. When the real part of any locus point goes positive, the system has achieved an unstable condition.

You might have noticed that the scrollbar can be operated by using the keyboard's up and down arrows. The arrows work after you have clicked the mouse on the scrollbar. Slide the scrollbar with the mouse until the highlighted poles get to a point of interest, then use the up and down arrows to "nudge" the scrollbar's slider.

Slide the scrollbar until the gain gets to GAIN = +2.1420E+01. This condition is shown in Figure 28.

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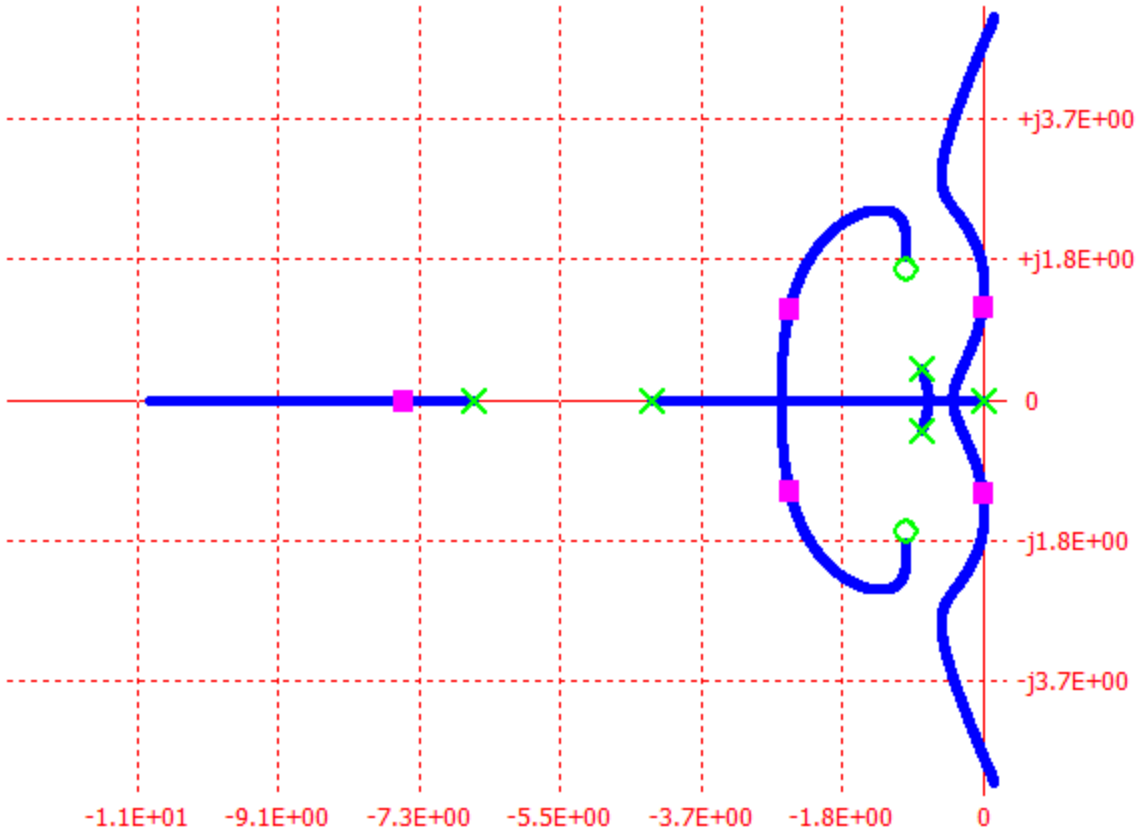

Figure 28. Example 2 with some of the highlighted poles in the right-half plane

This is the information in the right panel when the condition of Figure 28 is obtained:

```
MARKED LOCUS POINTS

GAIN = +2.1420E+01

REAL         IMAG
+9.0417E-05 +1.2047E+00
+9.0417E-05 -1.2047E+00
-2.5021E+00 +1.1828E+00
-2.5021E+00 -1.1828E+00
-7.4959E+00 +0.0000E+00
```

The real parts of two poles are slightly positive at +9.0417E-05 at a gain of 21.420.

Now use the scrollbar to increase the gain some more until the real parts of all poles are negative once again. This condition is shown in Figure 29.

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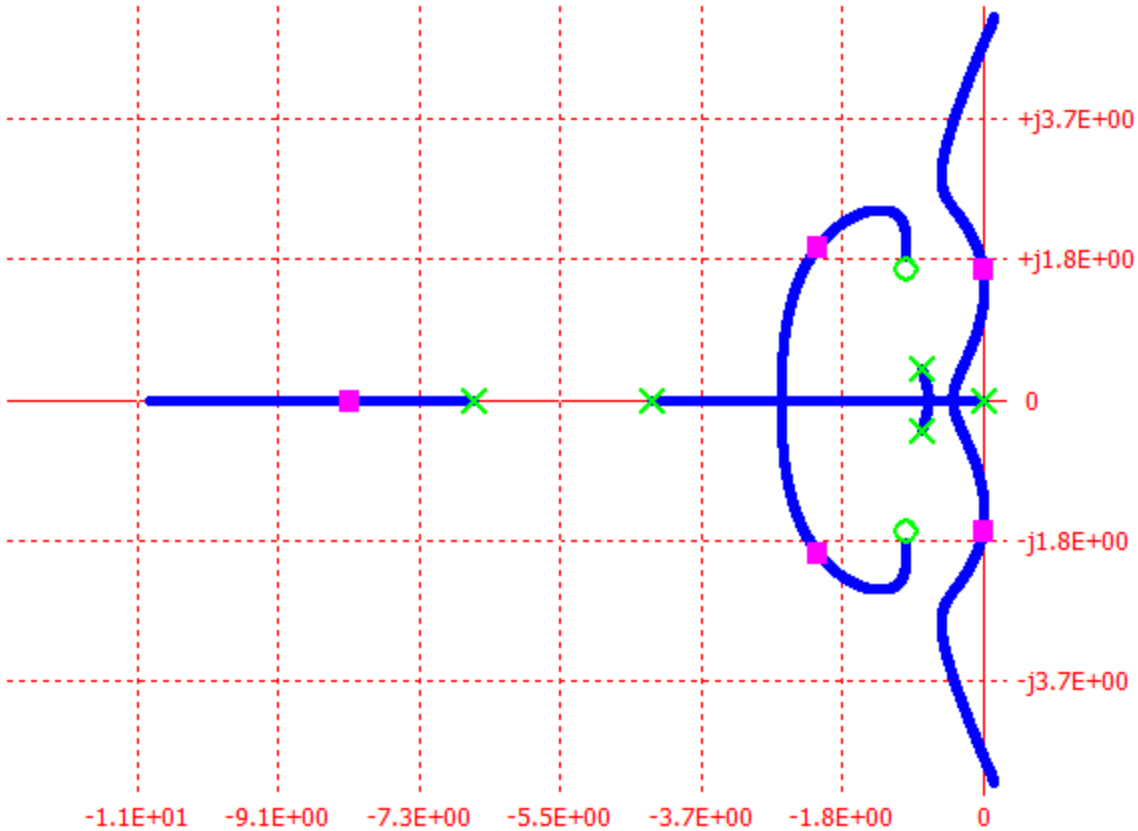

Figure 29. Example 2 with all highlighted poles in the left-half plane

This is the information in the right panel when the condition of Figure 29is obtained:

```
MARKED LOCUS POINTS

GAIN = +5.1569E+01

REAL         IMAG
-5.9094E-05 +1.6961E+00
-5.9094E-05 -1.6961E+00
-2.1478E+00 +1.9714E+00
-2.1478E+00 -1.9714E+00
-8.2043E+00 +0.0000E+00
```

The real parts of all poles are now all negative once again indicating a stable system. The poles that we have been watching now have a real value of -5.9094E-05 at a gain of 51.569.

Finally, increase the gain until some highlighted poles are again in the right-half plane. You should see the condition shown in Figure 30.

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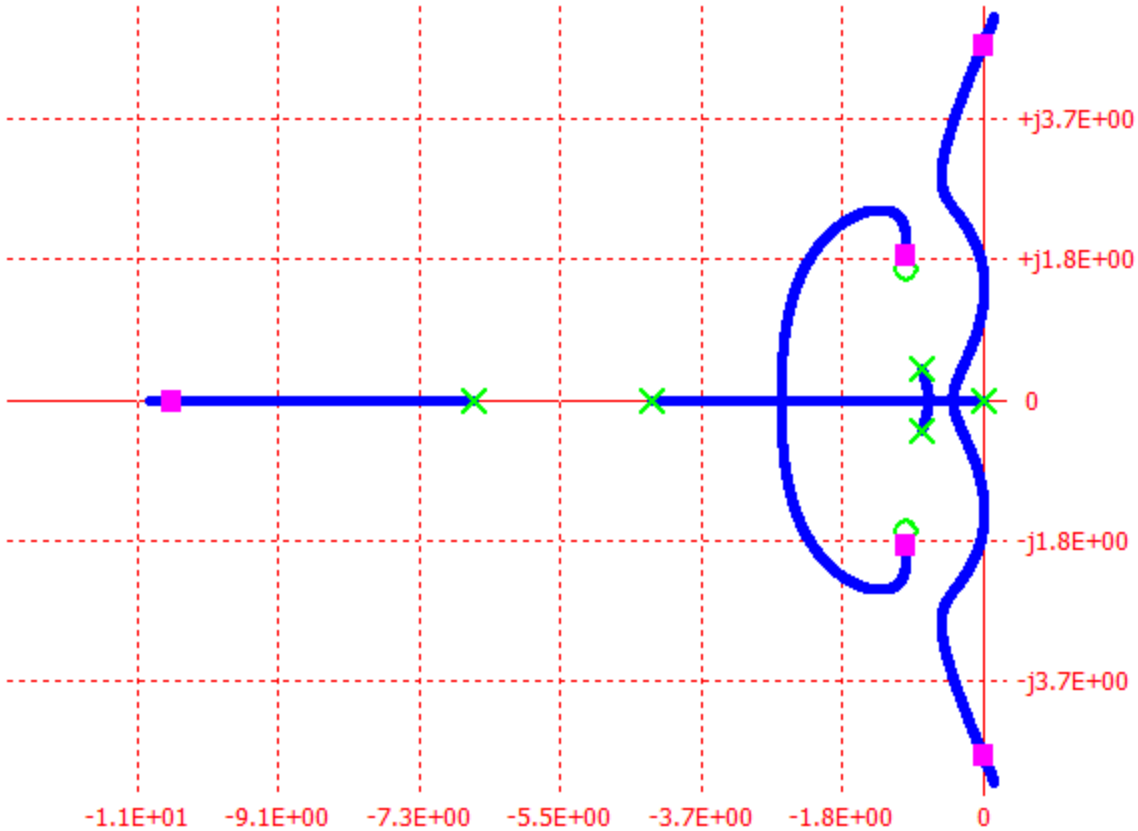

Figure 30. Example 2 with highlighted poles in the right-half plane once more

This is the information in the right panel for the condition of Figure 30:

```
MARKED LOCUS POINTS

GAIN = +2.5854E+02

REAL         IMAG
-9.9566E-01 +1.8752E+00
-9.9566E-01 -1.8752E+00
+1.7863E-03 +4.6069E+00
+1.7863E-03 -4.6069E+00
-1.0512E+01 +0.0000E+00
```

The real parts of the two poles of interest are now positive once again with real parts equal to +1.7863E-03 at a gain of 258.54. These poles will remain in the right-half plane as gain, K, approaches infinity.

So you have found that the example system is stable for some ranges of gain and unstable for other ranges of gain. That is, *the example system is conditionally stable.*

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Keep in mind that the plotted locus falls on discrete points. It's not continuous. So the stability and instability conditions found in this example are approximations.

Also, since the locus has a path that stays close to the imaginary axis until it finally moves forever into the right-half plane, the system is near a condition of instability under all conditions. It's unlikely that a system like this one would be very useful, robust, or reliable.

Notice one more thing. Because of the complicated nature of the locus in this example, it would difficult to plot it by hand. RootLocus has made the entire procedure easy.

<u>Step Response When Complex Poles are Near the jω Axis</u>

Return to the condition of Figure 29 by sliding the scrollbar until these conditions are reached once again:

```
MARKED LOCUS POINTS

GAIN = +5.1569E+01

REAL        IMAG
-5.9094E-05 +1.6961E+00
-5.9094E-05 -1.6961E+00
-2.1478E+00 +1.9714E+00
-2.1478E+00 -1.9714E+00
-8.2043E+00 +0.0000E+00
```

This condition causes a pair of complex poles to be located just to the left of the jω axis. That is, the system is very near a condition of instability. (Recall that complex poles on the jω axis are unstable because they will ring forever. A pair of complex poles in this example are very near the jω axis with σ = - 5.9094E-05.)

Click the Step Response button. You should see a step response plot like the one in Figure 31. With σ = - 5.9094E-05, the time constant is 1 / |σ| = 16922. (See the discussion of time constant on Page 46.) Such a large time constant implies a long settling time. The amplitude decay (to 0) of the waveform in Figure 31 is so slow that it is imperceptible in the figure.

When complex poles are near the jω axis, the system is at or near a condition of resonance (a natural inclination to ring at a particular frequency.) This condition is apparent in Figure 31.

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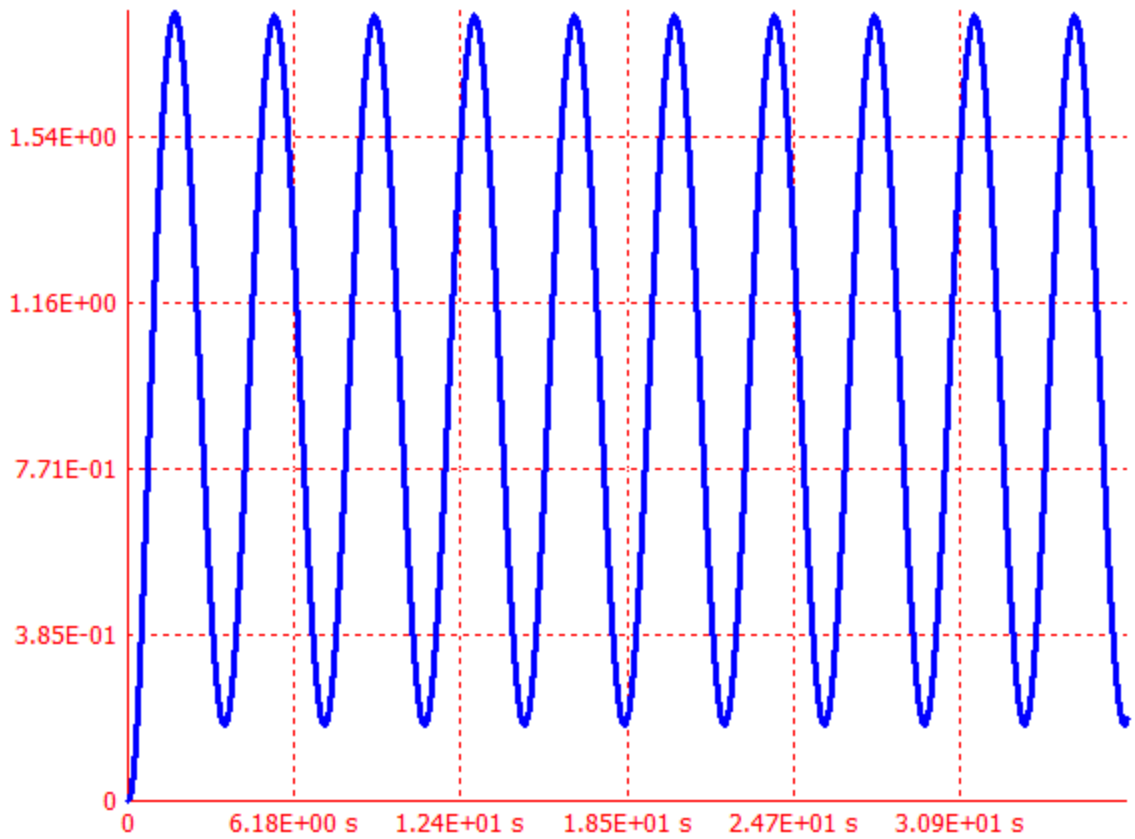

Figure 31. Step response of the system in Figure 29

Frequency Response When Complex Poles are Near the jω Axis

Given the conditions of the step response in the previous section, click the Freq Response button. You should see a frequency response plot like the one in Figure 32.

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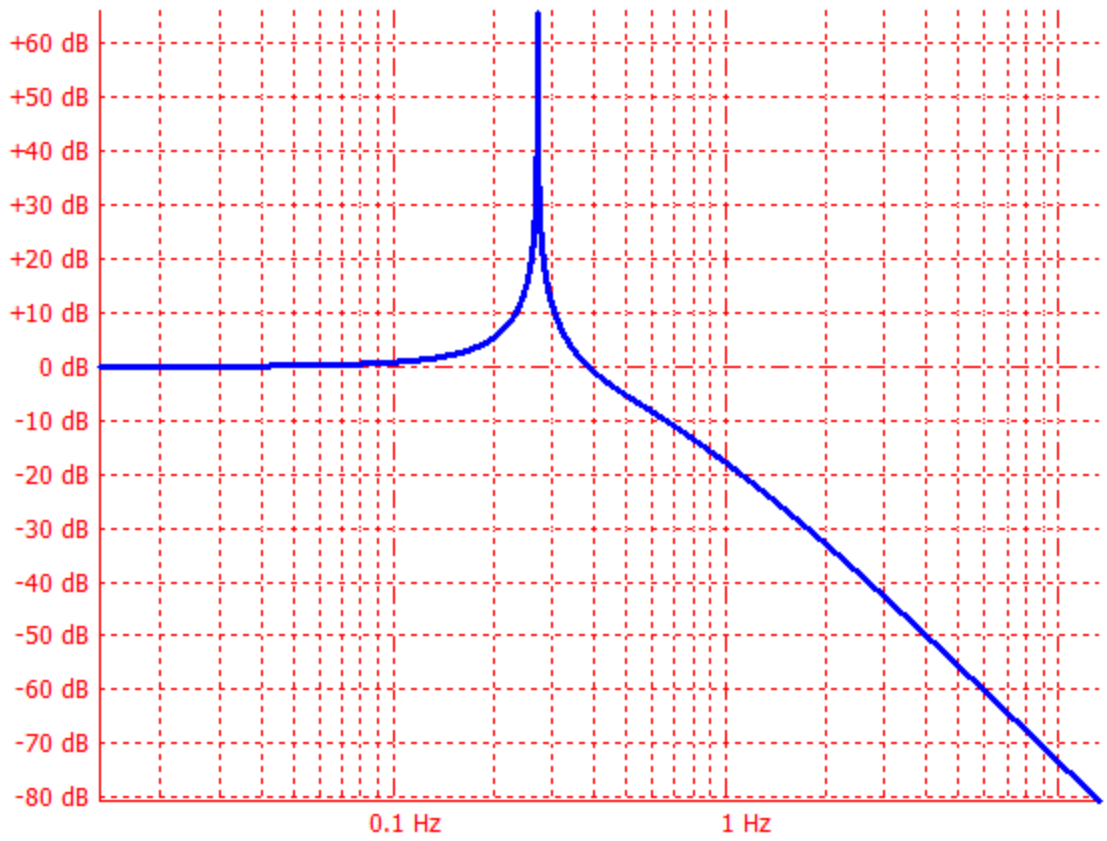

Figure 32. Frequency response of the system in Figure 29

The very narrow peaking response of Figure 32 is characteristic of a system exhibiting resonance or near-resonance. It's a sure sign that a system is at or near a condition of instability.

Sharp peaking in the frequency domain is a sure sign of ringing in the time domain. The narrower and higher the peak, the longer the duration of the ringing.

If you place the mouse pointer over the plot, you will obtain this response:

FREQUENCY RESPONSE: Maximum = +65.676 dB, Minimum = -80.638 dB, Initial = -0.012027 dB, Final = -80.638 dB

The frequency response calculations of RootLocus use an input amplitude of 1, i.e., 0 dB. But you can see that the peak response is +65.676 dB, an amplitude increase of 1922.

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Errors in the Estimated Maximum Frequency Response

The maximum amplitude just found is +65.676 dB. RootLocus' assessment of this value is based upon numerical calculations taken at arbitrary sample frequencies. Since the function's skirts are very steep near the peak, any offset from the true maximum response frequency produces a large error.

An analytical solution for the maximum amplitude shows that the peaking reaches about +93.10 dB at a frequency of about 0.2698995 Hz. Thus, you can see that RootLocus was wrong by more than 27 dB in this case!

RootLocus used a sample frequency of 0.269999 Hz to find its result of +65.6756 dB. So a frequency delta of only 0.269999 Hz - 0.2698995 Hz = 0.0000995 Hz leads to an error of more than 27 dB. When the frequency response exhibits a lot of peaking, errors of this type are inevitable.

Care must be taken when using the calculated maximum (or minimum) frequency response when that response has a sharp peaking characteristic.

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EXAMPLE 3: Using Zooming and Default Gain

Open RootLocus and enter these values:

Numerator coefficients:	8.26458e6 8.78525E5
Denominator coefficients:	136350.0 1.3761661e7 1.26662e7 10000.0 0
Maximum gain:	2e4
Minimum gain:	0
Plotting density:	Ultra Fine

Once again, notice the zero at the end of the list of denominator coefficients. Always remember to include zero coefficients where they exist. A zero coefficient can occur at any location except the high-order coefficient.

RootLocus lets you load all of the values listed above by selecting *Examples > Example 3* from the menu. This choice also computes the example root locus automatically.

Click the Calculate button. The root locus is calculated and the DEFAULT GAIN window shows a value of 60.612981. This is the ratio of the high-order numerator coefficient to the high-order denominator coefficient. That is, the default gain is the gain of the given system.

RootLocus should look like Figure 33. This plot is almost useless. One thing that you can see right away, however, is that the poles migrate into the right-half plane.

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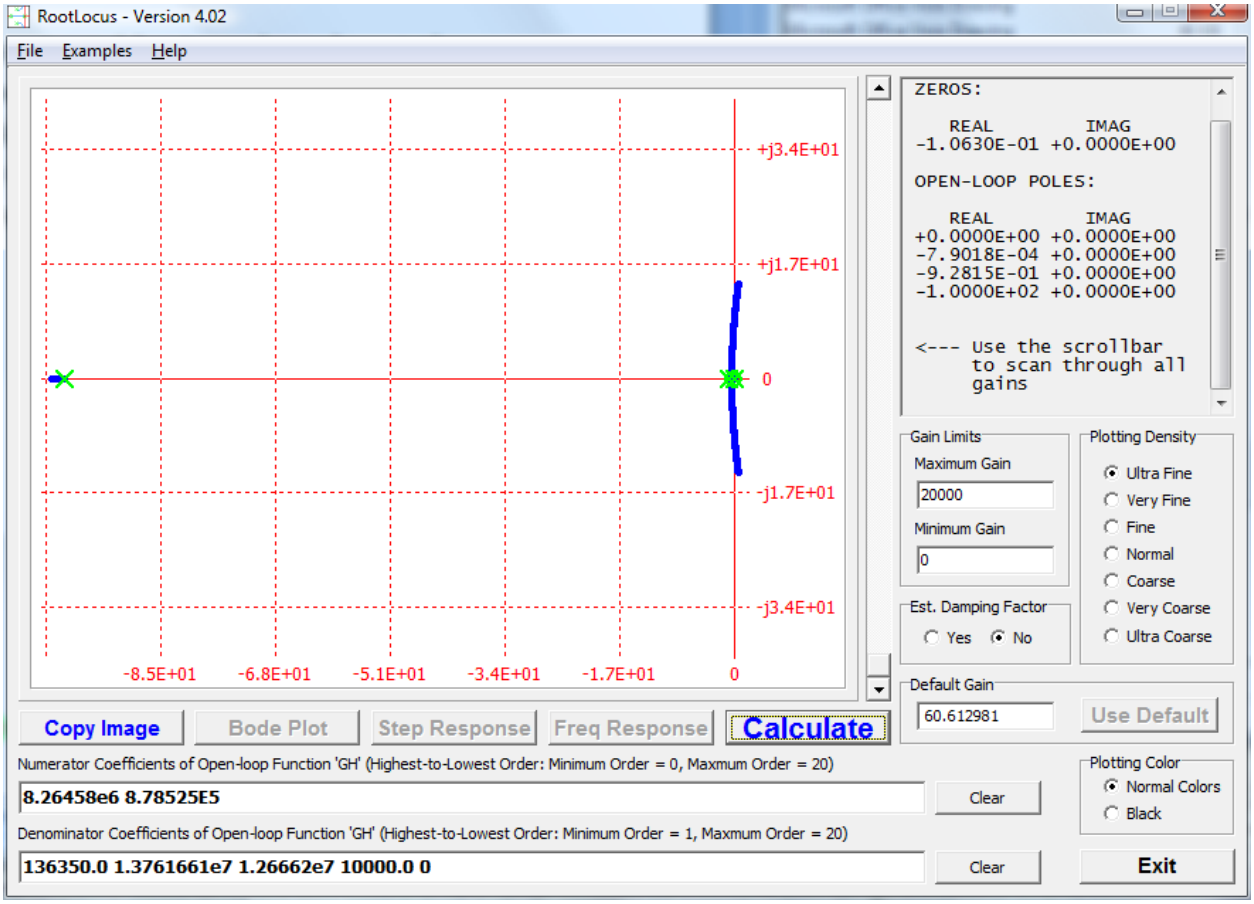

Figure 33. Example 3 after clicking Calculate

This information is contained in the right panel.

```
ZEROS:

   REAL         IMAG
-1.0630E-01 +0.0000E+00

OPEN-LOOP POLES:

   REAL         IMAG
+0.0000E+00 +0.0000E+00
-7.9018E-04 +0.0000E+00
-9.2815E-01 +0.0000E+00
-1.0000E+02 +0.0000E+00
```

You can see from the list that one real open-loop pole is located at 0 + j0, the origin. Another real open-loop pole is located at -100 + j0. Because RootLocus includes the entire calculated locus when it plots the root locus, the pole at s = -100 + j0 causes the scaling to squeeze all of the other parts of the locus into an area that's too small to allow the details of the locus to be visible.

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Because the pole at s = -100 + j0 is significantly farther from the imaginary axis than the other poles and the zero, it has very little affect on system performance. Therefore, for the purpose of evaluating system performance, it can be ignored.

Zooming is useful in cases like this one. For this example, place the mouse at about s = -3.5E+01 + j1.7E+01 and drag down to the right until the bottom side of the focus box (the box that's formed when dragging) just passes the grid line at -j1.7E+01. Release the mouse button. You should have a plot that looks something like Figure 34.

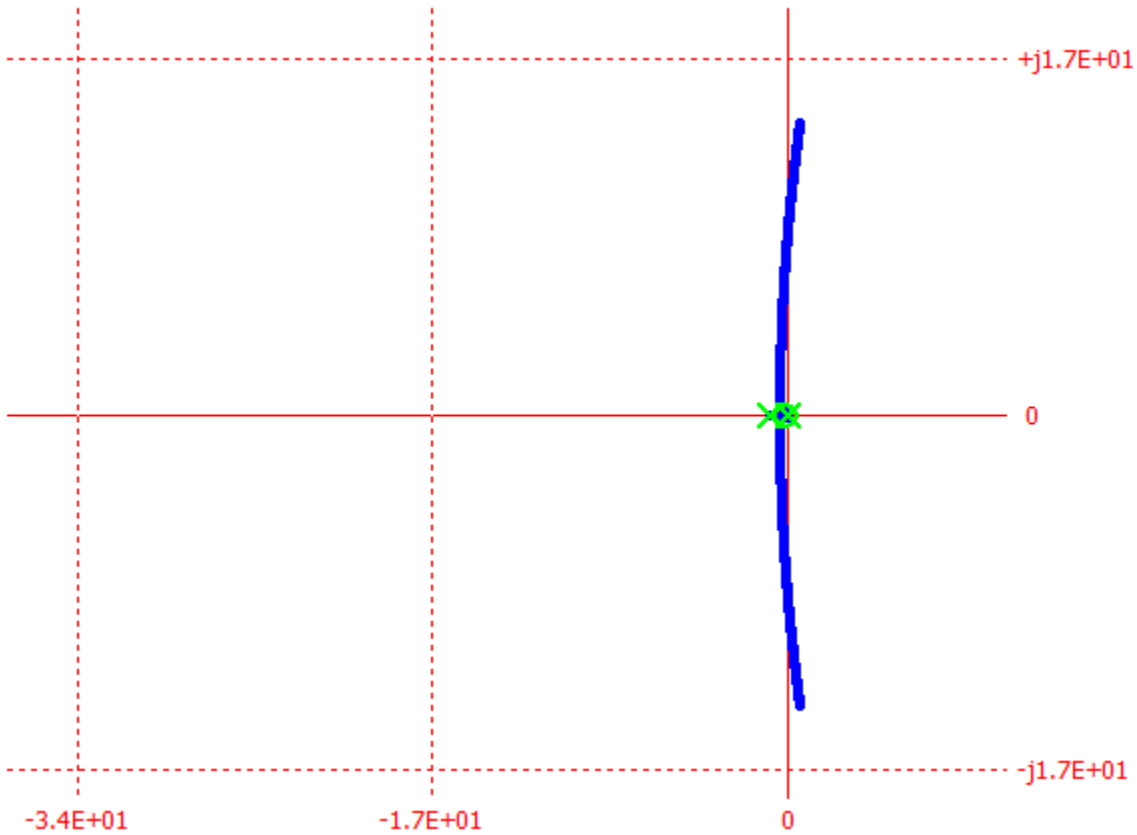

Figure 34. Example 3 after zooming

Now drag the scrollbar slider until the real part of one or more highlighted poles becomes positive. This will happen at a gain of GAIN = +8.2980E+03 = 8298.0 if you had been sure to select a PLOTTING DENSITY of Ultra Fine.

This is the gain that causes instability when the open-loop numerator and denominator polynomials have been normalized to 1.0. (See Default Gain on page 27 for an example of this normalization.)

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But what if the system of this example was an actual system? That is, suppose you, as a control system engineer, were presented with this system and asked to figure out just how stable it really is. Click Calculate again.

Now drag the mouse diagonally to zoom in on the information very near the origin. This area is identified by the green zero and poles. You might get a zoomed result like the one shown in Figure 35.

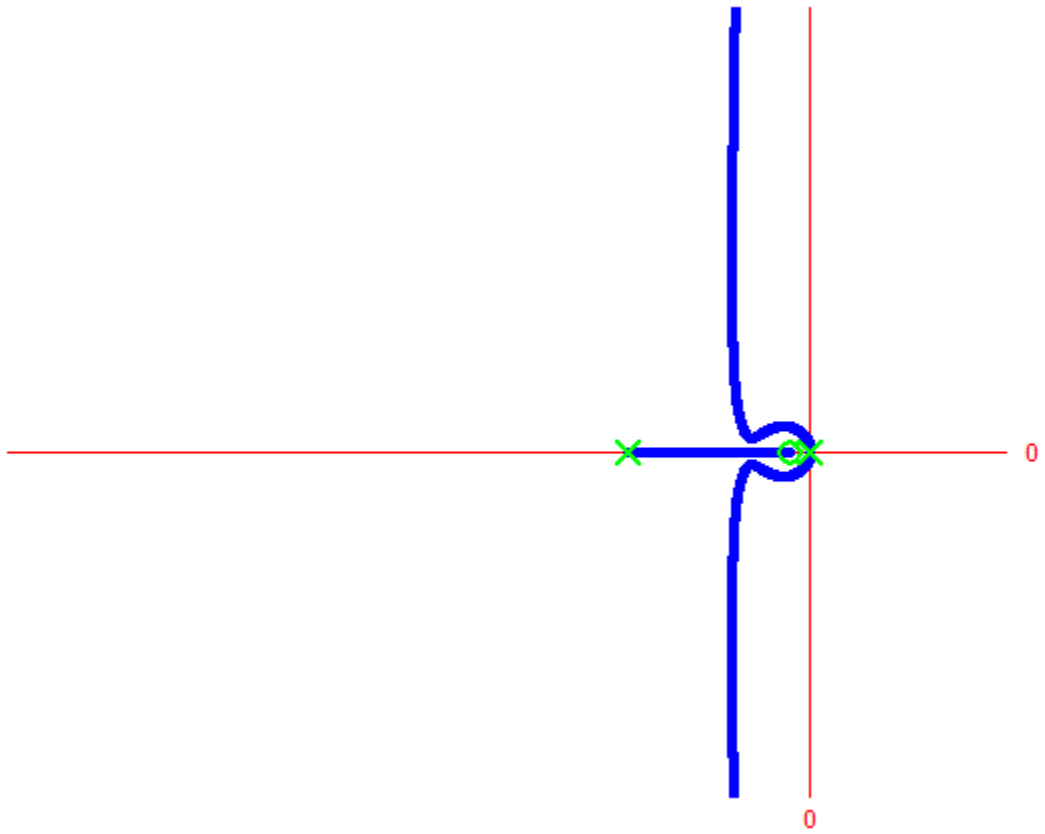

Figure 35. Example 3 after a lot of zooming

The result of zooming in Figure 35 shows that a certain amount of interesting detail would have been missed if you had not zoomed in on the plot. Zooming can be very useful when one or more unimportant poles or zeros cause the original, scaled plot to lose detail.

Given the condition in Figure 35, move the scrollbar slider so that highlighted poles appear. The Use Default button now will be enabled.

Click the Use Default button. Use Default will now be red. Also, the highlighted poles will be located at the default gain; i.e., the gain under MARKED LOCUS

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POINTS will be the same as the gain in the DEFAULT GAIN window. The highlighted locus points are shown in Figure 36.

It's clear from Figure 36 that the given system is stable since the poles at the default gain are all in the left-half plane. It would require a substantial increase in gain to make the system unstable. Since instability occurs at a gain of about 8298.0, instability would occur when the gain is 8298.0 / 60.612981 = 136.9 times larger than the default gain of the given system. You can safely report that the given system is stable.

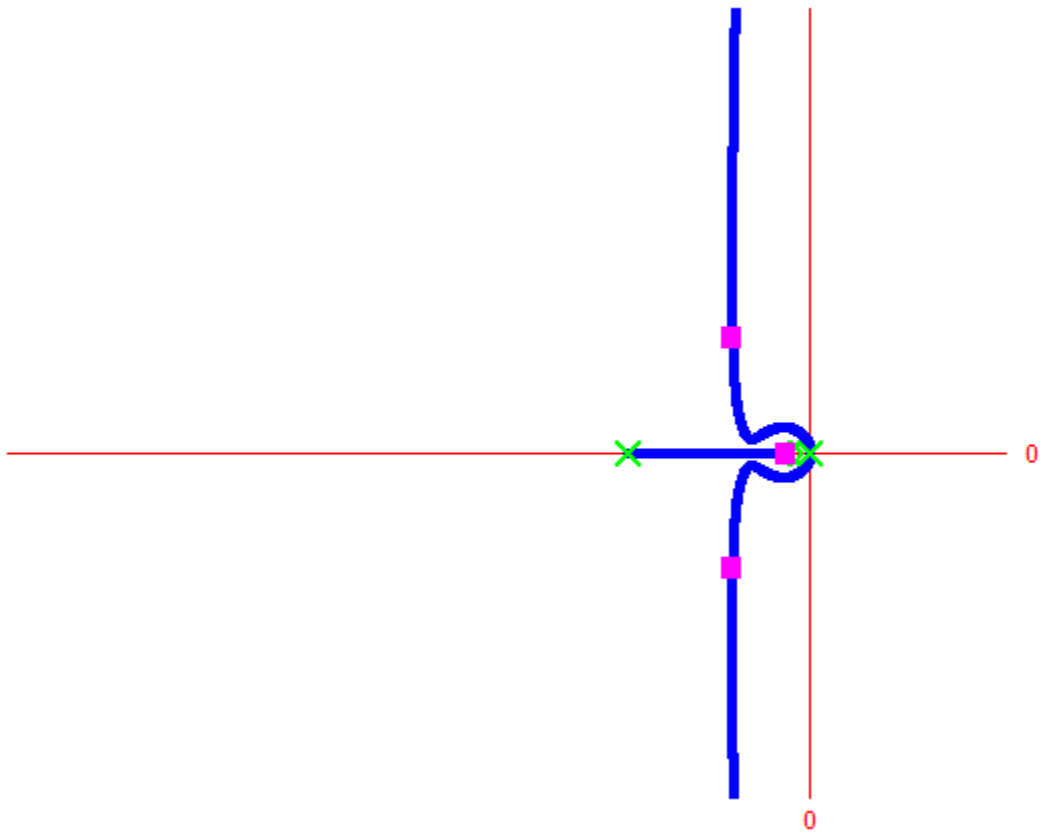

Figure 36. Highlighted poles represent the default gain of 60.612981 after clicking Use Default

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EXAMPLE 4: Using the Bode Plot Function to Find Gain Margin and Phase Margin

A Bode plot reveals system stability by using parameters called the gain margin and phase margin.

> **A system is unstable if the feedback signal is returned in phase with the input signal and the total loop gain is at least 1 (i.e., 0 dB).**

The gain margin and phase margin measure how far away a system is from this critical condition. This example will demonstrate gain margin and phase margin.

Open RootLocus and enter these values:

Numerator coefficients:	1
Denominator coefficients:	1 2 1 0
Maximum gain:	3
Minimum gain:	0
Plotting density:	Normal

Click the Calculate button to obtain the root locus plot. Slide the scrollbar to reach these conditions:

```
MARKED LOCUS POINTS

GAIN = +3.3426E-01

REAL         IMAG
-2.6205E-01 +3.9725E-01
-2.6205E-01 -3.9725E-01
-1.4759E+00 +0.0000E+00
```

The result should look like Figure 37. *Since the highlighted poles are all in the left-half plane, the system is unquestionably stable.*

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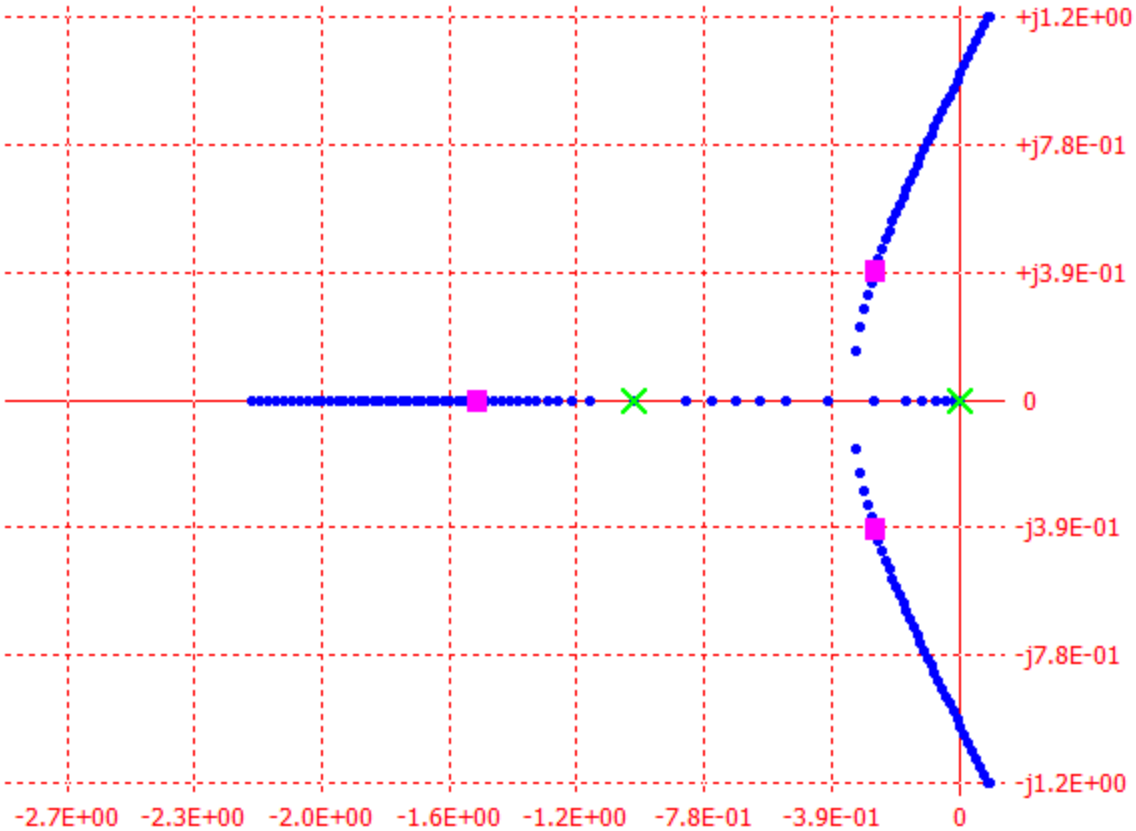

Figure 37. Example 4 after adjusting the scrollbar

<u>Gain Margin</u>

Click the Bode Plot button. Put the mouse pointer over the plot and move the cursor until these conditions are reached:

```
OPEN-LOOP
BODE PARAMETERS

FREQ = +0.1588 Hz
DB   =   -15.5 dB
DEG  = +0.1245 deg
```

These results are the closest you can move the cursor to a 0° condition.

Gain margin is the distance that the amplitude response is below 0 dB when the phase response is at 0°.

The amplitude response for the current condition is illustrated in Figure 38. Its value is -15.5 dB when the phase response is 0° as listed in the right panel. For this

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condition, we say that "The system has 15.5 dB of gain margin." This means that the loop gain is 15.5 dB below the level that would cause instability (oscillation) when the feedback signal is in phase with the input signal. *As a rule of thumb, a gain margin of at least 10 dB is considered acceptable for stability.*

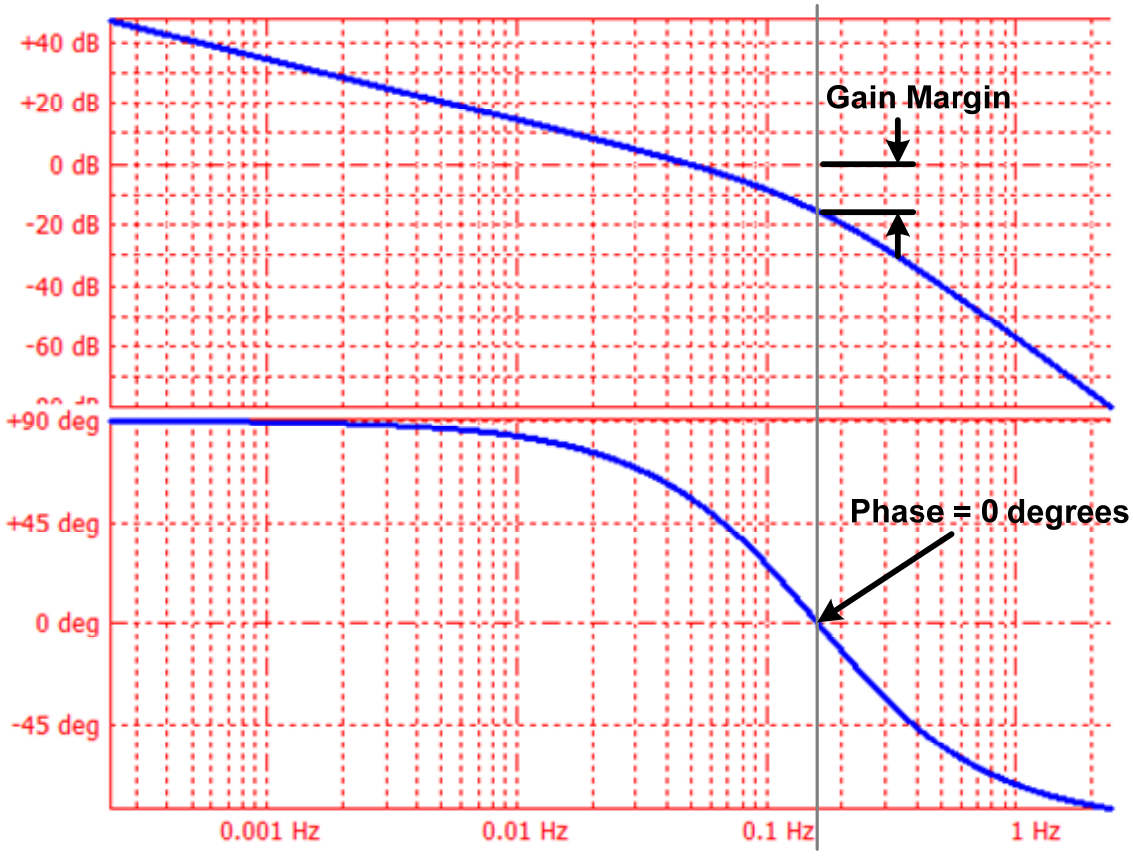

Figure 38. Example 4 gain margin

If the amplitude response is 0 dB or higher when the phase response is 0°, the system is unstable since the feedback signal is in phase with the input signal and at or above the input level that caused the feedback.

<u>Phase Margin</u>

Now put the mouse pointer over the plot and move the cursor until these conditions are reached:

```
OPEN-LOOP
BODE PARAMETERS

FREQ = +0.04885 Hz
DB   = -0.04149 dB
DEG  =  +55.87 deg
```

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These results are the closest you can move the cursor to a 0-dB condition.

Phase margin is the distance that the phase response is above 0° at the point where the amplitude response goes to 0 dB.

The phase margin for the current condition is illustrated in Figure 39. Its value is 55.87° as identified in the right panel. For this condition, we say that "The system has 55.87° of phase margin." This means that the feedback signal is 55.87° away from being in phase with the input signal when the feedback amplitude is at a level 0 dB. *As a rule of thumb, a phase margin of at least 45° is considered acceptable for stability.*

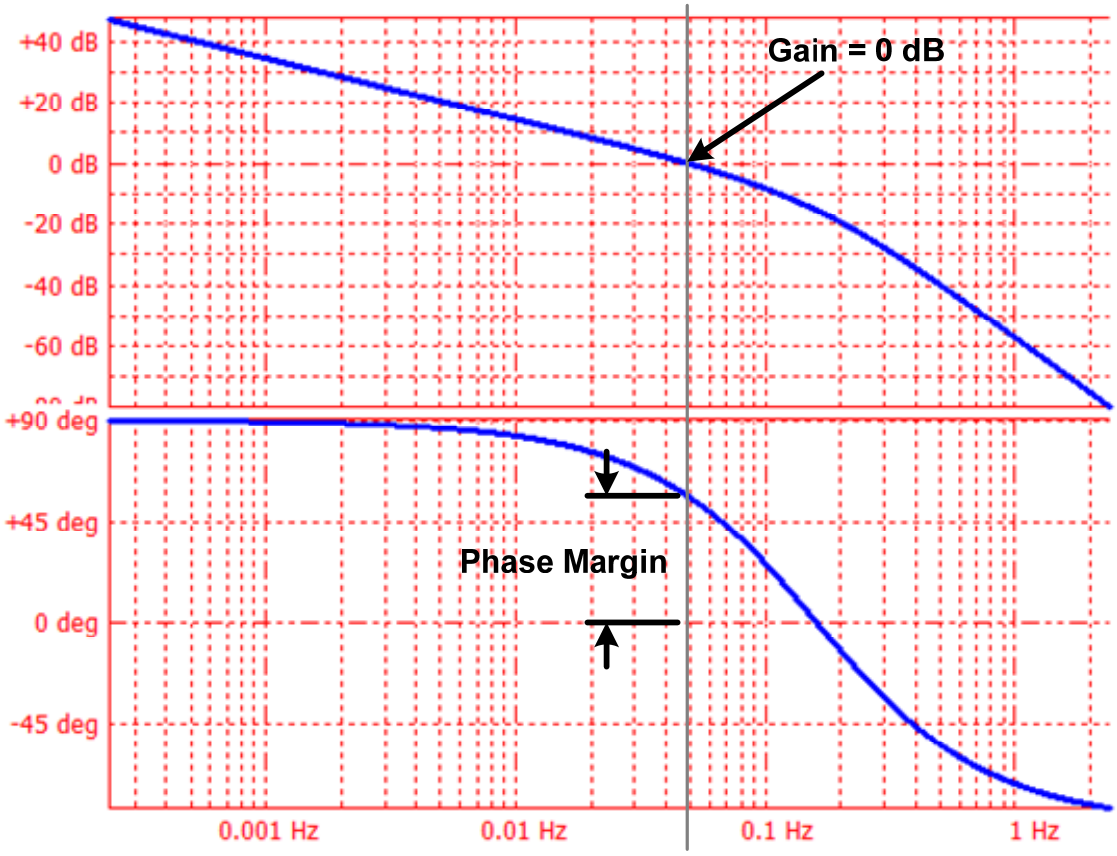

Figure 39. Example 4 phase margin

If the phase response is 0° or less when the amplitude response is 0 dB, the system is unstable since the feedback signal is at or past the phase shift needed to be in phase with the input signal at a signal level of 0 dB.

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Bode Responses for an Unstable System

Click the red Bode Plot button to return to normal mode. Slide the scrollbar until these conditions are reached:

```
MARKED LOCUS POINTS

GAIN = +3.0000E+00

REAL        IMAG
+8.7280E-02 +1.1713E+00
+8.7280E-02 -1.1713E+00
-2.1746E+00 +0.0000E+00
```

The system is now unquestionably unstable since two of the highlighted poles are in the right-half plane.

Click the Bode Plot button.

Place the mouse pointer over the Bode plot and move the cursor to the place where the phase response curve crosses through 0°. Notice that the amplitude response is greater than 0 dB. The means there is no gain margin. Since there is no gain margin, the Bode plot agrees with the root locus: the system is unstable.

Now move the cursor to the place where the amplitude response crosses through 0 dB. Notice that the phase response is below 0°. This mean there is no phase margin. Since there is no phase margin, the Bode plot agrees with the root locus: *the system is unstable.*

This example illustrates the advantage of the root locus over the Bode plot. The Bode plot shows that the system is unstable *for the selected gain*. The root locus shows that the system is unstable for the entire range of gains that causes poles to be in the right-half plane.

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